7

Gravitation

After the invention of satellites, many countries launched many satellites one after another. Launching a satellite is a very complex process. Multiple satellites are loaded on a rocket and launched in their specific orbits. The launch of rockets and satellites requires knowledge of laws of motion, angular velocity, the force of gravitation, acceleration due to gravity and more. In this chapter, you will apply the concepts from previous chapters and learn about planetary forces.

Topic Notes

- *Universal Gravitation*
- *Gravitational Energy and Satellite Motion*

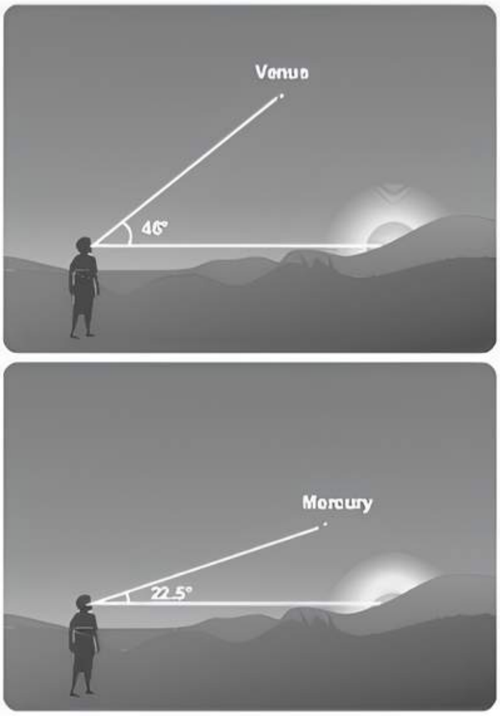
UNIVERSAL GRAVITATION 1

TOPIC 1

KEPLER'S LAWS

Kepler's laws of planetary motion, in astronomy and classical physics, describe the motions of the planets in the solar system. They were derived by the German astronomer Johannes Kepler, whose analysis of the observations of the 16th-century Danish astronomer Tycho Brahe enabled him to announce his first two laws in the year 1609 and a third law nearly a decade later, in 1618. Kepler himself never numbered these laws or especially distinguished them from his other discoveries.

For example, to calculate the distances of Mercury and Venus from the Sun as shown in the figure.



The maximum angular distance Venus and Mercury can subtend at a location on Earth with regard to the Sun is 46 degrees for Venus and 22.5 degrees for Mercury, as they are inner planets with respect to Earth. When Venus reaches its greatest elongation (46 degrees) with regard to Earth, it forms a 90-degree angle with the Sun. We can now calculate the distance between Venus and the Sun. One astronomical unit equals the distance between the Earth and the Sun (1 AU).

The trigonometric relation satisfied by this right-angled triangle is $\sin\theta = \frac{r}{R}$

Here, $\sin 46° = 0.77$.

Hence, Venus is at a distance of 0.77 AU from the Sun. Similarly, the distance between Mercury (θ is 22.5 degrees) and Sun is calculated as 0.38 AU. To find the distance of exterior planets like Mars and Jupiter, a slightly different method is used.

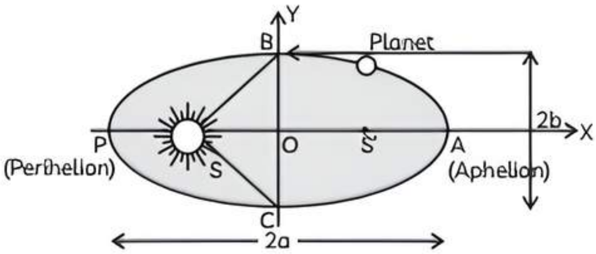
Using this concept, Kepler introduced his laws and this helps in understanding the reason behind the formulation of Kepler's law.

Kepler's Laws of Planetary Motion

Kepler gave three laws of planetary motion.

Law of Orbits

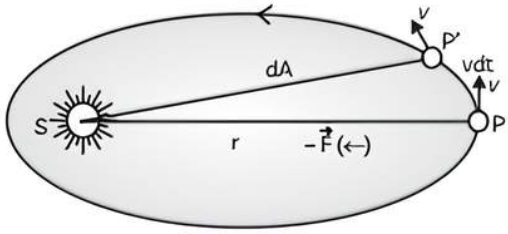
All planets revolve around the sun in elliptical orbits, with the Sun at one of its foci.



OP = OA = Semi-major axis

Law of Areas

The line that joins any planet to the sun sweeps equal areas in equal intervals of time



$$\frac{dA}{dt} = \text{constant}$$

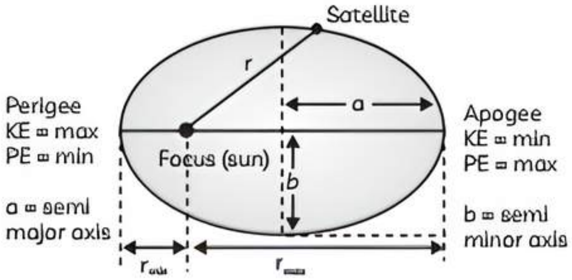
$$= \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv = \frac{L}{2m}[\text{since, } L = mvr]$$

Law of Periods

The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet. Considering that the satellites/planets revolve in an elliptical orbit, the orbital radius can be equivalent to the semi-major axis of the ellipse traced out by the satellite.

i.e. $T^2 \propto a^3$, or, $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$

The position of a planet nearest to the sun is known as perigee. In this position, the speed of the planet is maximum. The position of a planet at the maximum distance from the sun is known as apogee. In this position, the speed of the planet is minimum.



Universal Law of Gravitation

The discovery of the law of Gravitation

The way by which the law of universal gravitation was discovered is often considered the paradigm of modern scientific technique. The major steps involved were:

(1) The hypothesis about planetary motion given by Nicolas Copernicus (1473-1543).

(2) The careful experimental measurement of the positions of the planets and the sun by Tycho Brahe (1546-1601).

(3) Analysis of the data and the formulation of empirical laws by Johannes Kepler (1571-1630).

(4) The development of a general theory by Isaac Newton (1642-1727).

Newton's law of Gravitation

It states that every particle in the universe attracts all other particles with a force which is directly proportional to the product of their masses and is inversely proportional to the square between them.

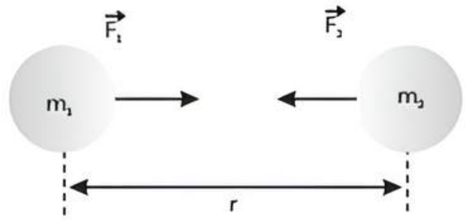
If $|\vec{F}_{12}| |\vec{F}_{21}| = F$,

then $F \propto m_1 m_2$ and $F \propto \frac{1}{r^2}$

So $F \propto \frac{m_1 m_2}{r^2}$

$$F = \frac{Gm_1 m_2}{r^2}$$

[G = universal gravitational constant]



This formula is applies only to spherically symmetric masses or point masses.

Vector form of Newton's law of gravitation

Let $\vec{r}_{12}$ = position vector of m_1 w.r.t m_2 = $\vec{r}_1 - \vec{r}_2$

$\vec{r}_{21}$ = position vector of m_2 w.r.t m_1 = $\vec{r}_2 - \vec{r}_1$

$\vec{r}_{21}$ = gravitational force exerted on m_2 by m_1

$\vec{r}_{12}$ = gravitational force exerted on m_1 by m_2

$\vec{F}_{12} = \frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{12}$, $\vec{F}_{21} = \frac{Gm_1 m_2}{r_{21}^3}\hat{r}_{21}$

Negative sign shows that:

(1) The direction of F_{12} is opposite of that of $\hat{r}_{21}$

(2) The gravitational force is attractive in nature

(3) Similarly, $\vec{F}_{21} = \frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21}$

or $\vec{F}_{21} = \frac{Gm_1 m_2}{r_{12}^2}\hat{r}_{21}$

$\Rightarrow$ $\vec{F}_{12} = -\vec{F}_{21}$, $\vec{r}_{12} = -\vec{r}_{21}$

The gravitational force between two bodies is equal in magnitude and opposite in direction.

Universal Gravitational constant

Universal gravitational constant is a scalar quantity.

Value of G = 6.67×10^{-11} N-m^2/kg^2;

CGS : G = 6.67×10^{-8} dyne-cm^2/gm^2

Dimension: $[M^{-1} L^3 T^{-2}]$

Its value is the same throughout the universe. G does not depend on the nature and size of the bodies; it does not depend even upon the nature of the medium between the bodies.

Its volume was first found out by the scientist "Henry Cavendish" with the help of "Torsion Balance" experiment.

Example 1.1: Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. assuming that the only forces acting on the particles are their mutual gravitation. Find the initial acceleration of the heavier particle.

Ans. Force extended by one particle on another is

$$F = \frac{Gm_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 2}{(0.5)^2}$$

$$= 5.34 \times 10^{-10} \text{ N}$$

Acceleration of heavier particles.

We know that, $F = ma$

$$a = \frac{F}{m} = \frac{5.3 \times 10^{-10}}{2}$$
$$= 2.67 \times 10^{-10} m/s^2$$

Related Theory

➥ *This example shows that gravitational force is quite weak but this is the only force that binds our solar system and also the universe consisting of all galaxies and other interstellar systems.*

Example 1.2: Two stationary particles of masses M_1 and M_2 are "d" distance apart. A third particle lying on the line joining the particles experiences no resultant gravitational force. What is the distance of this particle from M_1?

Ans. Let m be the mass of the third particle.

Force on m towards M_1 is $F_1 = \frac{GM_1 m}{r^2}$

Force on m towards M_2 is $F_2 = \frac{GM_2 m}{(d-r)^2}$

Since net force on m is zero $\therefore F_1 = F_2$

$$\Rightarrow \quad \frac{GM_2 m}{r_2} = \frac{GM_1 m}{(d-r)^2}$$

$$\Rightarrow \quad \left(\frac{d-r}{r}\right)^2 = \frac{M_1}{M_2}$$

$$\Rightarrow \quad \frac{d}{r} - 1 = \frac{\sqrt{M_1}}{\sqrt{M_2}}$$

$$\Rightarrow \quad r = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}\right]$$

Example 1.3: Let us assume that our galaxy consists of 2.5×10^{11} stars each with one solar mass. How long will this star at a distance of 50,000 light years from the galactic center take to complete one revolution? Take the diameter of the Milky Way to be 10^5 light years. $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. [NCERT]

Ans. Given, $r = 50{,}000$ ly $= 50,000 \times 9.46 \times 10^{15}$

$$m = 4.73 \times 10^{20} m$$
$$M = 2.5 \times 10^{11} \text{ solar mass}$$
$$= 2.5 \times 10^{11} \times 2 \times 10^{30} \text{ kg}$$
$$= 5 \times 10^{41} \text{kg}$$

$$M = \frac{4\pi^2 r^3}{GT^2}$$

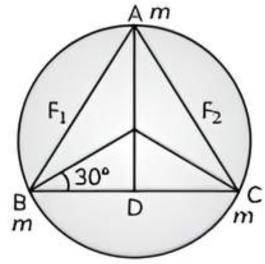
$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{1/2}$$

$$= \left[\frac{4 \times (3.14)^2 (4.73 \times 10^{20})^3}{6.67 \times 10^{-11} \times 5 \times 10^{41}}\right]^{1/2}$$

$$= 1.12 \times 10^{16} \text{ s}$$

Example 1.4: Three particles, each of the mass m, are situated at the vertices of an equilateral triangle of the side "a". The only forces acting on the particles are their mutual gravitational forces. It is intended that each particle moves along a circle while maintaining its original separation "a". Determine the initial velocity that should be given to each particle and the time period of the circular motion.

Ans.



The resultant force on a particle at A due to other particles is

$$F = \sqrt{F_{AB}^2 + F_{AB}^2 + 2F_{AB}F_{AB}\cos 60°}$$

$$= \sqrt{3}\frac{Gm^2}{a^2} \quad \left(\because F_{AB} = F_{AC} = \frac{Gm^2}{a^2}\right)$$

Radius of the circle of $r = \frac{a}{\sqrt{3}}$

If each particle is given, a tangential velocity v, so that the resultant force acts as the centripetal force.

Then, $\frac{mv^2}{r} = \sqrt{3}\frac{mv^2}{a}$

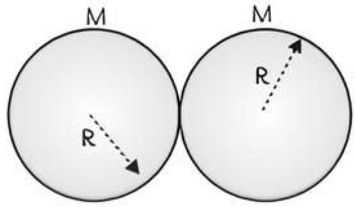
$$\sqrt{3}\frac{mv^2}{a} = \frac{Gm^2\sqrt{3}}{a^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{Gm}{a}}$$

$$T = \frac{2\pi r}{v}$$
$$= \frac{2\pi a}{\sqrt{3}}\sqrt{\frac{a}{Gm}}$$
$$= 2\pi\sqrt{\frac{a^3}{3Gm}}$$

Example 1.5: Two solid spheres of the same size of a certain metal are placed in contact with each other. Prove that the gravitational force acting between them is directly proportional to the fourth power of their radius.

Ans. The weights of the sphere may be assumed to be concentrated at their centers.



So, $F = \dfrac{G\left(\frac{4}{3}\pi r^3\rho\right)\times\left(\frac{4}{3}\pi R^3\rho\right)}{(2R)^2} = \frac{4}{9}\left(G\pi^2\rho^2\right)R^4$

$\therefore \quad F \propto R^4$

Example 1.6: Two identical point masses, each of mass 1 kg lie in the (*x-y-z*) plane at point (0, 0) (0, 0, 0.2 m) and (0, 0.2 m, 0) respectively. The gravitational force on the mass at the origin is,

(a) $1.94 \times 10^{-11}(\hat{i}+\hat{j})$ N

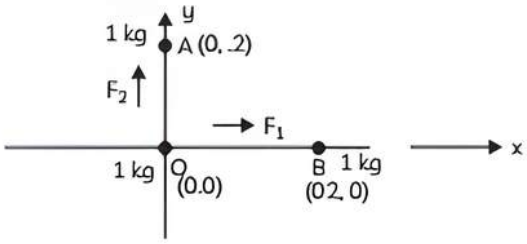
(b) $3.34 \times 10^{-10}(\hat{i}+\hat{j})$ N

(c) $1.67 \times 10^{-9}(\hat{i}+\hat{j})$ N

(d) $3.34 \times 10^{-10}(\hat{i}+\hat{j})$ N

Ans. *(c) $1.67 \times 10^{-9}(\hat{i}+\hat{j})$ N*

Explanation:



Gravitational force, $F = \dfrac{Gm_1m_2}{r^2}$

Given: $r = 0.2\ m$

$m_1 = m_2 = 1$ kg

Now force on an object at O due to B,

$$F_1 = \frac{6.67\times10^{-11}(1)(1)}{(0.2)^2}\hat{i} = 1.67\times10^{-9}\hat{i}$$

Force on an object at O due to A.

$$F_2 = \frac{6.67\times10^{-11}(1)(1)}{(0.2)^2}\hat{j} = 1.67\times10^{-9}\hat{j}$$

Hence net force, $F = 1.67 \times 10^{-9}\,\hat{i}+\hat{j}$ N.

Acceleration Due to Gravity

In a class of 50 students, the teacher planned an activity to see how an object accelerates. She asked students to pick something up with their hand and drop it. When you release it from your hand, its speed is zero and on the way down its speed increases. The longer it falls the faster it travels. Its acceleration is more than just increasing speed. Now Pick up this same object and toss it vertically into the air. On the way up its speed will decrease until it stops and reverses direction. Decreasing speed is also considered acceleration. But acceleration is more than just changing speed. Now again she asked to pick up your battered object and launch it one last time. This time throw it horizontally and notice how its horizontal velocity gradually becomes more and more vertical. Since acceleration is the rate of change of velocity with time and velocity is a vector quantity, this change in direction is also considered acceleration.

In each of these examples, acceleration was the result of gravity. Your object was accelerating because gravity was pulling it down. Even the object tossed straight up is falling — and it begins falling the minute it leaves your hand. If it wasn't, it would have continued moving away from you in a straight line. This is the acceleration due to gravity.

Gravity

In Newton's law of gravitation, the force of attraction between any two bodies is gravitation. If one of the bodies is earth then gravitation is called gravity. Hence, gravity is the force by which the earth attracts a body towards its center. It is a special case of gravitation.

Acceleration Due To Gravity Near Earth's Surface

Let us assume that the earth is a uniform sphere of mass M and radius R. The magnitude of the gravitational force of the earth on a particle of mass *m*, located outside the earth at a distance *r* from its center is

$$F = \frac{GMm}{r^2}$$

Now according to Newton's second law $F = ma$

$$\text{Therefore, } a = \frac{GM}{r^2}$$

At the surface of the earth, acceleration due to gravity

$$g = \frac{GM}{R^2} = 9.8\ m/s^2$$

However, any value of *g* measured at a given location will differ from the *g* value calculated according to the equation due to three reasons

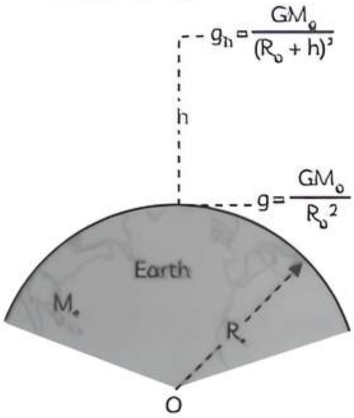
(1) earth's mass is not distributed uniformly

(2) Earth is not a perfect sphere

(3) earth rotates

Variations in Acceleration due To Gravity

Due To Altitude (Height)



From diagram

$$\frac{g_h}{g} = \frac{R_e^2}{(R_e+h)^2} = \frac{R^2}{R_e^2\left[1+\frac{h}{R_e}\right]} = \left(1+\frac{h}{R_e}\right)^{-2}$$

By binomial expression $\left(1+\frac{h}{R_e}\right)^{-2} = \left(1-\frac{2h}{R_e}\right)$

if $h << R_o$, then higher power terms become negligible

$$\therefore \quad g_h = g\left[1-\frac{2h}{R_e}\right]$$

Due to Depth

Assuming that the density of the earth remains the same throughout the volume.

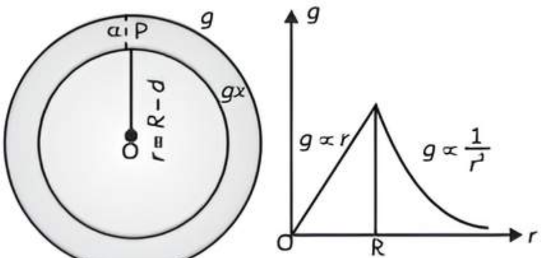
At earth's surface: $g = \frac{4}{3\pi G R_e \rho}$

At depth d inside the earth:

For point P only the mass of the inner sphere is effective

$$g_d = \frac{4}{3\pi G R_e \rho}$$

Mass of sphere radius r = M'



(Taking direction towards centre of earth as positive)

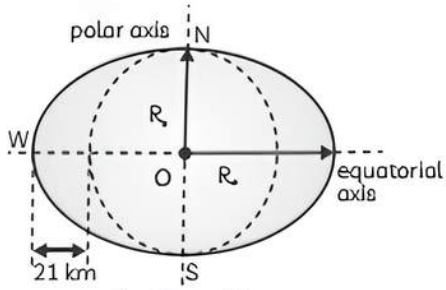
$$M = \frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi r^3 \times \frac{M_e}{\frac{4}{3}\pi r R^3} = M' = \frac{M_E}{R^3}r^3$$

$$g_d = \frac{G}{r^2} \times \frac{M_e r^3}{R_e^3} = \frac{GM_e}{R_e^2} \times \frac{r}{R_e} = \frac{GM_e}{R_e^2} \times \frac{R_e - d}{R_e}$$

$$g_d = g\left[1-\frac{d}{R_e}\right] \text{ valid for any depth}$$

Due to Shape of the Earth

From the Diagram



(not to scale)

$R_p < R_e (R_e = R_p + 21 \text{ km})$

$$g_p = \frac{GM_e}{R_p^2}$$

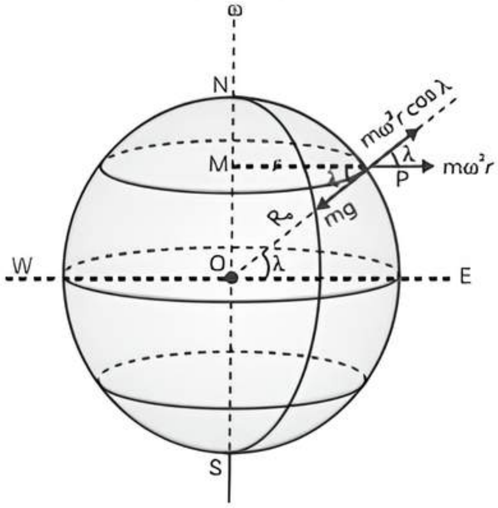
and $$g_e = \frac{GM_e}{(R_p + 21000)^2} = g_e < g_p$$

by putting the values $g_p - g_e = 0.02 \text{ m/s}^2$

Due to Rotation of the Earth

Net force on a particle at P.

$$Mg' = mg - mr\omega^2 \cos\lambda$$



$$G' = g - \omega r\omega^2 \cos\lambda$$

From, ΔOMP

$$r = R_o \cos\lambda$$

Where, λ = latitude

Substituting for r, we have

$$g' = g - R_o\,\omega^2 \cos^2\lambda$$

At the equator (λ – 0°):

$$g_{eq} = g - \omega^2 R \cos^2(0°)$$

At the poles ($\lambda - 90^o$)

$$g_{pole} = g - \omega^2 R\cos^2(90^o)$$
$$= g - \omega^2 R(0)$$
$$g_{max} \text{ or } g_{pole} = g$$

It means that acceleration due to gravity at the poles does not depend upon the angular velocity or rotation of the earth.

Condition of Weightlessness on Earth's Surface

If the apparent weight of a body is zero then the angular speed of the earth can be calculated as

$$mg' = mg\ mR_e\omega^2\cos^2\lambda$$

$$0 = mg - mR_e\omega^2\cos^2\lambda$$

$$\Rightarrow \omega = \frac{1}{\cos\lambda}\sqrt{\frac{g}{R_e}}$$

But at the equator $\lambda = 0°$

$$\therefore \omega = \sqrt{\frac{g}{R_e}} = \frac{1}{800}\text{rad} = 0.00125 \text{ rad/s}$$

$$= 1.25 \times 10^{-3} \text{ rad/s}$$

Important

➥ *If the earth rotates 17 times of its present angular speed, then bodies lying on the equator would fly off into space. Time period of the earth's rotation in this case would be 1.4h.*

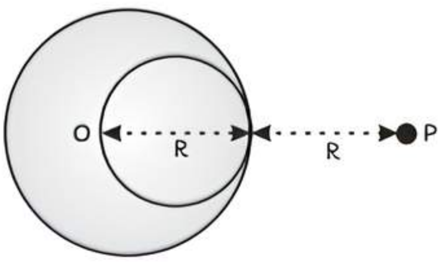
Example 1.7: Determine the speed with which the earth would have to rotate about its axis so that a person on the equator weighs 3/5th of its present value. Write your Answer in terms of g and R.

Ans. Weight on the equator, $W' = \frac{3}{5}W$

$$\Rightarrow \quad \frac{3}{5}mg = mg - m\omega^2 R$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2g}{5R}}$$

Example 1.8: A solid sphere of uniform density and radius R exerts a gravitational force of attraction F_1 on a particle P, distant 2R from the centre of the sphere. A spherical cavity of radius $\frac{R}{2}$ is not formed in the sphere as shown in figure. The sphere with cavity now applies a gravitational force F_2 on the same particle P. find the ratio $\frac{F_2}{F_1}$.



Ans.
$$F_1 = \frac{GMm}{4R^2}$$

F_2 = force due to the whole sphere – force due to the sphere-forming the cavity

$$= \frac{GMm}{4R^2} - \frac{GMm}{18R^2}$$

$$\Rightarrow \quad F_2 = \frac{7GMm}{36R^2}$$

$$\therefore \quad \frac{F_2}{F_1} = \frac{7}{9}$$

Example 1.9: Case Based:

In any space project, the mass of the space shuttle is carefully observed. The fuel in space shuttles is enough for them to enter Earth's atmosphere after which it falls under the effect of gravity. The free-falling space shuttle enters the atmosphere at such an angle that its nose and underside contract and compresses the air and absorb most of the heat generated. The fall of the space shuttle is so high that it often gets red hot. The underside of the space shuttle is incredibly heat resistant, insulating silica tiles.

(A) Assertion (A): The Square of the period of revolution of a planet is proportional to the cube of its distance from the sun.

Reason (R): Sun's gravitational field is inversely proportional to the square of its distance from the planet.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false

[Delhi Gov. QB 2022]

(B) The radius of the earth is about 6400 km and that of mars is 3200 km. The mass of the earth is 10 times that of mars. An object weighs 200 N on the surface of the earth. Its weight on the surface of mars will be:

(a) 80 N (b) 40 N
(c) 20 N (d) 8 N

(C) Weight of a body decreases by 1% when it is raised to a height h above the earth's surface. If the body is taken to a depth g in a mine. Then its weight will be:

(a) decrease by 0.5% (b) decrease by 2%
(c) increase by 0.5% (d) increase by 1%

(D) At which height from the earth's surface does the acceleration due to gravity decrease by 1%?

(E) Find the percentage decrease in the weight of a body when taken to a height of 16 km above the surface of the earth? (Radius of the earth is 6400 km)

Ans. (A) *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: By second equation of Newton we have,

By Kepler's law, $T_2 \propto R_3$

Where T is period of revolution

R is distance from sun.

Also gravitational field $\propto \left[\dfrac{1}{(\text{distance})^2}\right]$

Hence, both statements are correct but reason is not explanation of assertion.

(B) *(a) 80 N*

Explanation: Radius of earth $R_e = 6400$ and that of Mars is $R_m = 3200$

so $R_e = 2R_m$

And if the mass of earth is M_e and that of Mars is M_m then $M_e = 10M_m$

Value of g on mars $g_m = \dfrac{GM_m}{R_m^2}$

and that on earth is, $g_e = \dfrac{GM_e}{R_e^2}$

$$= \frac{G\times 10M_m}{4\times R_m^2}$$

$$= 2.5\,\frac{GM_m}{R_m^2}$$

or $g_m = \dfrac{g_e}{2.5}$

Weight on earth is 200 N.

$$\text{So, mass} = \frac{200}{g_e}$$

Now weight on Mars will be

$$\text{mass}\times g_m = \left(\frac{200}{g_e}\right)\times g_m$$

$$= 200\times\frac{g_m}{g_e}$$

$$= \frac{200}{2.5}$$

$$= 80\text{ N}$$

(C) *(a) decrease by 0.5%.*

Explanation: Percentage change in g when the body is raised to height h.

$$\frac{\Delta g}{g}\times 100\% = \frac{2h\times 100}{R} = 1\%$$

Percentage change in g when the body is taken into depth d.

$$\frac{\Delta g}{g}\times 100\% = \frac{d}{R}\times 100\% = \frac{h}{R}\times 100\%$$

[As $d = h$]

$\therefore$ Percentage decrease in weight

$$= \frac{1}{2}\left(\frac{2h}{R}\times 100\right)$$

$$= \frac{1}{2}(1\%) = 0.5\%.$$

(D) We know that

Acceleration due to gravity at height h from surface of the earth:

$$g' = g\left(1-\frac{2h}{R}\right)$$

$$\frac{g'}{g} = \left(1-\frac{2h}{R}\right)$$

$$\frac{99}{100} = \left(1-\frac{2h}{R}\right)$$

$$\frac{99}{100} = 1-\frac{2h}{R}$$

$$\frac{2h}{R} = 1-\frac{99}{100}$$

$$\frac{2h}{R} = \frac{1}{100}$$

$$h = \frac{6.4\times 10^6}{2\times 100}$$

$$h = 3.2\times 10^4$$

$$h = 32\text{ km.}$$

(E) Weight on the earth, $W = mg$.

Weight above the surface, $W' = mg'$

We know that, $g = 9.8\text{ m/s}^2$

$g' = ?$

$$g' = g\left(1-\frac{2h}{R}\right)$$

$$= g - \frac{2hg}{R}$$

$$g - g' = \frac{2hg}{R}$$

$$\%\text{ decrease in weight} = \frac{mg - mg'}{mg} \times 100$$

$$= \frac{g-g'}{g} \times 100$$

$$= \frac{2gh}{Rg} \times 100 = \frac{2h}{R} \times 100$$

$$= 2 \times \frac{16}{6400} \times 100$$

$$= \frac{1}{2}$$

$$= 0.5\%$$

Example 1.10: What is the value of acceleration due to gravity at a height equal to half the radius of the earth, from the surface of the earth? (Take $g = 10\ m/s^2$ on earth's surface).

Ans.

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = \frac{g}{\left(1+\frac{R_e}{2R_e}\right)^2} \qquad \left[\because h = \frac{R_e}{2}\right]$$

$$= \frac{g}{\left(1+\frac{1}{2}\right)^2} = \frac{g}{\left(\frac{3}{2}\right)^2} = \frac{4g}{9}$$

$$\therefore \quad g' = \frac{4g}{9}$$

$$g' = 4.44\ m/s^2.$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Newton's law of gravitation:

(a) is not applicable outside the solar system.

(b) is used to govern the motion of satellites only.

(c) control the rotational motion of satellites and planets.

(d) control the rotational motion of electrons in atoms

Ans. *(c) control the rotational motion of satellites and planets.*

Explanation: According to Newton's law of gravitation, every particle attracts other particles in the universe by a force which is directly proportional to the product of their masses and inversely proportional to the square of displacement between them.

$$F \propto \frac{m_1 m_2}{r^2}$$

Here, force is:

(1) Attractive

(2) Independent of the medium between them.

Hence, it controls the rotational motions of satellites and planets.

$$\frac{GmM_e}{d^2} = \frac{mv^2}{d}$$

Related Theory

➥ *Kepler's laws are stated for planets orbiting the sun, they are valid for all bodies satisfying the two previously stated conditions.*

2. Gravitational force between two masses of a distance '*d*' apart is 6 N. If these masses are taken to the moon and kept at the same separation, then the force between them becomes:

(a) 1 N (b) 16 N

(c) 36 N (d) 6 N

Ans. *(d) 6 N*

Explanation: If the masses are taken to the moon and kept at the same separation, *i.e.*, the distance is *d* then, Force will remain the same *i.e.*, 6 N. It is just because we know that the value of G is constant throughout the universe including the moon and the masses do not change from place to place.

Related Theory

➥ *The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.*

3. A body is responded on a spring balance in a ship sailing along the equator with speed *v*. If ω is the angular speed of the earth and W_0

is the scale reading when the ship is at rest, the scale reading when the ship is sailing is:

(a) W_0 (b) zero

(c) $W_0\left(1 \pm \frac{2\omega v}{g}\right)$ (d) $W_0\left(1 - \frac{g}{2\omega}\right)$

[Delhi Gov. QB 2022]

Ans. *(c)* $W_0\left(1 \pm \frac{2\omega v}{g}\right)$

Explanation: When the ship is at rest:

$$g_o = g - \omega^2 R \quad \text{...(i)}$$

When the ship is moving with velocity v' from west to east

$$\omega' = \omega_o + \omega_b$$

When the ship is moving east to west

$$\omega' = \omega_o - \omega_b$$

On combining the equations we can write:

$$\omega' = \omega_o \pm \omega_b$$

$$\Rightarrow \quad \omega' = \omega \pm \frac{v'}{R}$$

g effective of the ship then becomes:

$$g_o' = g - R\left(\omega \pm \frac{v'}{R}\right)^2$$

$$\Rightarrow \quad g_o' = g - R\left(\omega^2 \pm \frac{2\omega v'}{R} + \frac{v'^2}{R^2}\right)$$

The last term is very small and can be neglected

$$\Rightarrow \quad g_o' = g - R\left(\omega^2 \pm \frac{2\omega v'}{R}\right)$$

$$g_o' = g - R\omega^2 \pm 2\omega v'$$

From eqn (i)

$$g_o' = g_o \pm 2\omega v'$$

$$g_o' = g_o\left(1 \pm \frac{2\omega v'}{g_o}\right)$$

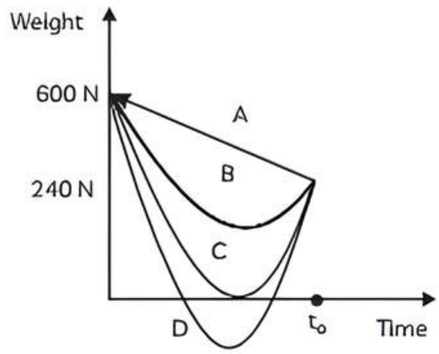
multiplying both sides by the mass m and using weight $W = mg$, we get,

$$\Rightarrow \quad W' = W_0\left(1 \pm \frac{2\omega v'}{g_o}\right)$$

Related Theory

➥ *Although Newton's law of gravitation applies strictly to the particles, we can also apply it to real objects as long as the sizes of the objects are very small relative to the distance between them. It is universally valid and applies from small objects on the earth to plants, stars and even galaxies.*

4. Suppose the acceleration due to gravity at the earth's surface is 10 m/s^2 and at the surface of mars it is 4.0 m/s^2. A 60 kg passenger goes from the earth to mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of the figure best represents the weight (net gravitational force) of the passenger as a function of time?



(a) A (b) B

(c) C (d) D

[NCERT Exemplar]

Ans. *(c) C*

Explanation: Since the acceleration due to gravity varies inversely to the square of the distance. Hence, the apparent weight (net gravitational force) of the passenger with respect to time will not be a straight line but a curve. In between the earth and mars, there will be a point where the gravitational force due to both of the bodies will be equal and opposite, hence the apparent weight will be zero but never negative. Out of the three curves in the figure, only curve C fulfills this condition.

5. Mars has a diameter of approximately 0.5 of that of earth and a mass of 0.1 of that of earth. The surface gravitational field strength on mars as compared to that on earth is a factor of:

(a) 0.1 (b) 0.2

(c) 2.0 (d) 0.4

Ans. *(d) 0.4*

Explanation: As we know

$$g = \frac{GM_e}{R_e^2}$$

$$g_{mars} = \frac{G(0.1)M_e}{(0.5)^2 R_e^2}$$

$$= \frac{0.4GM_e}{R_e^2} = 0.4g$$

⚠ Caution

➥ *Students should know that this formula is valid if h is up to 5% of the earth's radius. (320 km from the earth's surface)*

If h is greater than 5% of the earth's radius we use

$$g_h = \frac{GM_e}{(R_e + h)^2}$$

6. On some planet, 'g' is 1.96 m/s^2. If it is safe to jump from a height of 2 m on earth, then what should be the corresponding safe height for jumping on that planet?

(a) 5 m (b) 2 m
(c) 10 m (d) 20 m

Ans. *(c) 10 m*

Explanation: At earth, $g_e = 9.8$ m/s^2, $h_e = 2$ m

At another planet, $g_p = 1.96$, $h = ?$

Use $mg_1h_1 = mg_2h_2$

$\Rightarrow$ $1.96\,(h_1) = (9.8)\,(2)$

$h_1 = 10$ m

7. Gravitation on the moon is $\frac{1}{6}$th of that on earth. When a balloon filled with hydrogen is released on the moon then, this:

(a) will rise with an acceleration less than $\frac{g}{6}$

(b) will rise with acceleration $\frac{g}{6}$

(c) will fall down with an acceleration less than $\frac{5g}{6}$

(d) will fall down with acceleration $\frac{g}{6}$

Ans. *(d) will fall down with acceleration $\frac{g}{6}$*

Explanation: There is no atmosphere on the moon, meaning no buoyant force in an upward direction, so it will fall down with acceleration $\frac{g}{6}$.

8. When the radius of the earth is reduced by 1% without changing the mass, then the acceleration due to gravity will:

(a) increase by 2% (b) decrease by 1.5%
(c) increase by 1% (d) decrease by 1%

Ans. *(a) increase by 2%*

Explanation: Given that, $\frac{\Delta R}{R} = 1\%$,

The acceleration due to gravity

$$g = \frac{GM}{R^2}$$

$$\frac{\Delta g}{g}\% = \frac{-2\Delta R}{R}$$

As, $$\frac{\Delta g}{g}\% = -2(-1)\% = +2\%.$$

The maximum permissible error in the acceleration due to gravity.

9. Acceleration due to gravity at the earth's surface is 'g' m/s^2. Find the effective value of acceleration due to gravity at a height of 32 km from sea level: (R = 6400 km)

(a) 0.5 g m/s^2 (b) 0.99 g m/s^2
(c) 1.01 g m/s^2 (d) 0.90 g m/s^2

Ans. *(b) 0.99 g m/s^2*

Explanation: For height h, above the earth's surface

$$g_{eff} = g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{32\times2}{6400}\right) = g(1-0.01)$$

$$g_{eff} = 0.99 \text{ g m/s}^2$$

Related Theory

➥ *The acceleration due to gravity is maximum at the earth's surface. It decreases whether we go at higher altitudes or we move below the surface.*

10. Imagine a new planet having the same density as that of the earth but its radius is 3 times bigger than the earth's size. If the acceleration due to gravity on the surface of the earth is g and that on the surface of the new planet is 'g' then

(a) g' = 3g (b) g' = g/9
(c) g' = 9g (d) g' = 27g

Ans. *(a) $g' = 3g$*

Explanation: The acceleration due to gravity:

$$g = \frac{GM}{R^2}, M = \frac{4}{3}\pi R^3\rho$$

$$g = \frac{G\times\frac{4}{3}\pi R^3\rho}{R^2}$$

$$g = G\times\frac{4}{3}\pi R\rho$$

$$g \propto R$$

$$\frac{g'}{g} = \frac{R'}{R}$$

$$\frac{g'}{g} = \frac{3R}{R}$$

$$\frac{g'}{g} = 3$$

$$g' = 3g$$

$$g' \propto g$$

11. Four particles of masses m, $2m$, $3m$ and $4m$ are kept in sequence at the corners of a square of side a. the magnitude of

gravitational force acting on a particle of mass m placed at the center of the square will be:

(a) $\frac{24m^2G}{a^2}$ (b) $\frac{6m^2G}{a^2}$

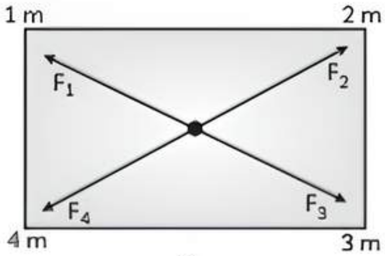
(c) $\frac{4\sqrt{2}Gm^2}{a^2}$ (d) zero [Diksha]

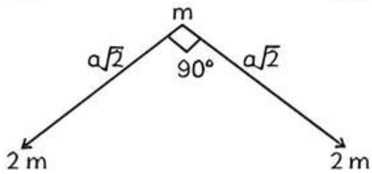
Ans. (c) $\frac{4\sqrt{2}Gm^2}{a^2}$

Explanation: If two particles of mass m are placed a distance apart then the force of attraction

$$G\frac{mm}{a^2} = F \text{ (Let).}$$

Now according to the problem particle of mass m is placed at the center of the square. Then it will experience four forces.





The net force at the center is,

$$F_{net} = F_1 + F_2 + F_3 + F_4$$

$$= \frac{Gmm}{x^2} + \frac{G2mm}{x^2} + \frac{G3mm}{x^2} + \frac{G4mm}{x^2}$$

$$= 2\sqrt{2}F$$

$$F_{net} = 2\sqrt{2}F$$

$$= 2\sqrt{2}\frac{FG \times 2m \times 2}{(a/\sqrt{2})^2} = \frac{4\sqrt{2}Gm^2}{a^2}$$

12. A uniform ring of mass M and radius R is placed directly above a uniform sphere of mass 8 m and same radius R. The center of ring is at a distance of $d = \sqrt{3}$ R from the center of the sphere. The gravitational attraction between the sphere and ring is:

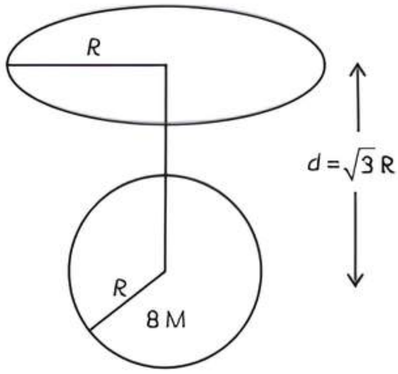
(a) $\frac{GM^2}{R^2}$ (b) $\frac{3GM^2}{R^3}$

(c) $\frac{2GM^2}{\sqrt{2}R^2}$ (d) $\frac{\sqrt{3}GM^2}{R^2}$

[Delhi Gov. QB 2022]

Ans. (d) $\frac{\sqrt{3}GM^2}{R^2}$

Explanation:



Gravitational field due to ring at a distance $d = \sqrt{3}\,R$ on its axis is

$$E = \frac{GMd}{(R^2 + d^2)^{\frac{3}{2}}} = \frac{GM\sqrt{3}R}{(R^2 + 3R^2)^{\frac{3}{2}}}$$

$$= \sqrt{3}\frac{GMR}{8R^3} = \frac{\sqrt{3}GM}{8R^2}$$

Therefore, Force on the sphere = Mass of the sphere × E

Force on the sphere = 8ME = $\frac{\sqrt{3}GM^2}{R^2}$

13. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity:

(a) will be directed towards the centre but not the same everywhere.

(b) will have the same value everywhere but not directed towards the centre.

(c) will be same everywhere in magnitude directed towards the centre.

(d) cannot be zero at any point.

[NCERT Exemplar]

Ans. (d) *cannot be zero at any point.*

Explanation: If the density of the earth is non-uniform then 'g' at different points will be different $\left(\therefore g = \frac{4}{3}\pi\rho GR\right)$ So, the $g = 0$ cannot be any point.

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

14. Assertion (A): The difference in the value of acceleration due to gravity at the pole and equator is the proportional to the square of angular velocity of the earth.

Reason (R): The value of acceleration due to gravity is minimum at the equator and maximum at the pole.

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: Acceleration due to gravity,

$$g' = g - R_o\omega^2\cos^2\lambda$$

At equator, $\lambda = 0°$,

$\therefore$ $\cos 0° = 1$

$\therefore$ $g_o = g - R_o^2$

At poles, $\lambda = 90°$,

$\therefore$ $\cos 90° = 0$

$g_p = g$

Thus, $g_p - g_o = g - g + R_o\omega^2 = R_o\omega^2$

Also, the value of g is maximum at the poles and minimum at the equator.

15. Assertion (A): The comets do not obey Kepler's Laws of planetary motion.

Reason (R): The comets do not have elliptical orbits.

[Delhi Gov. QB 2022]

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The orbit of a comet is not elliptical. As a result, it does not obey Kepler's rule of planetary activity. They travel on hyperbolic or nearly parabolic routes.

16. Assertion (A): If the rotation of the earth its axis suddenly stops then the acceleration due to gravity will increase at all places on the earth.

Reason (R): At height h from the surface of the earth. Acceleration due to gravity is:

$$g_h = g\left(1 - \frac{2h}{R_e}\right)$$

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: The acceleration of gravity is caused by the gravitational force that the earth exerts on things, but because the world rotates, various centrifugal forces impinge on objects at different latitudes, hence the acceleration of gravity varies. If the earth stops rotating, the centrifugal force disappears and just the gravitational force occurs, and the acceleration due to gravity becomes the same everywhere.

17. Assertion (A): A body falling freely under the force of gravity has constant acceleration (9.81 m/sec^2).

Reason (R): Earth attracts everybody towards its center by the same force.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The gravity of the Earth is the total acceleration that is imparted to the objects due to the combined effect of gravitation and the centrifugal force. It is denoted by "g" and is approximately equal to 9.8 m/s^2 (meter per Second Square).

Gravity is the force by which the planet or any other body draws the objects towards its center and therefore the force of gravity keeps all of the planets in orbit around the Sun. Any object which has mass has gravity. Objects with more mass have more gravity and those with less mass have less gravity. Also, the gravity gets weaker with distance and therefore, the closer objects to each other have the stronger gravitational pull. Earth's gravity comes from all its mass which makes the combined gravitational pull on all the mass in the body that gives us the weight. We exert the same gravitational force on the Earth that the Earth does on us. Since the Earth is more massive than us, our force does not work on it. The Earth is one massive object with very strong gravitational force. As a result, the gravity of Earth attracts all objects towards the center of Earth.

18. Assertion (A): Kepler's law of areas is equivalent to the law of conservation of angular momentum.

Reason (R): For planetary motion $\frac{dA}{dt} = \frac{L}{2m}$ = Constant.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: According to Kepler's law,

$$\frac{dA}{dt} = \text{constant}$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

$$\frac{mr^2\omega}{2m} = \frac{L}{2m}$$

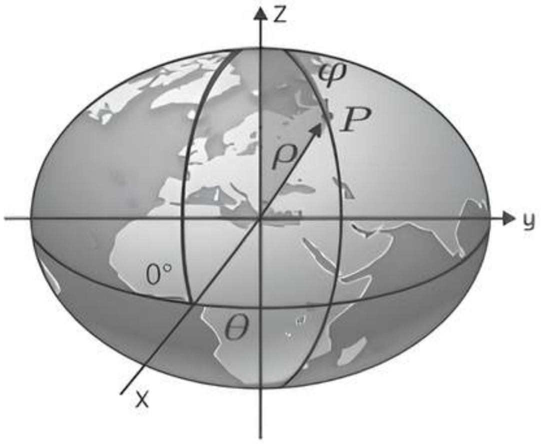
$$L = \text{constant}$$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

19. *The earth is not a perfect sphere. It is an ellipsoid. The radius in the equatorial plane is 21 km larger than the radius along the poles. Due to this, the gravity is more at poles and less at the equator.*



Both, due to rotation and the bulging at the equator, the value of the g is smaller at the equator than the poles. The surface of the planet plays an important role in the value of gravity. The sun does not revolve around hence it is nearly spherical. The mass of the sun causes it to have the greatest value of gravity in our solar system.

(A) **At what altitude above the earth's surface does the value of '*g*' equal that of a 100-kilometer-deep mine?**

(B) **At which height from the earth's surface does the acceleration due to gravity decrease by 75% of its value at the earth's surface?**

(C) **The commodity weights as per its real weight at equators, whereas it differs at poles, making a product more cost-effective to purchase at the equator than at the poles. Why there is a difference in weighing at both places?**

Ans. (A) Let the acceleration due to gravity at height h be the same as that at a depth d, deep into the earth i.e., d = 100 km.

Therefore,

$$g' = g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$$

$$\frac{2h}{R} = \frac{d}{R}$$

$$2h = d$$

$$h = \frac{d}{2} = 50 \text{ km}$$

(B) Since, the acceleration due to gravity reduces by 75%, then the value of acceleration due to gravity there is $g' = 100 - 75 = 25\%$.

It means, $g' = \frac{25}{100} g.$

If h is the height of location above the surface of the earth, then

$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$$

$$\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2} = \frac{25}{100}$$

$$= \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$1+\frac{h}{R} = 2$$

$$R + h = 2R$$

$$H = R$$

Therefore, at R = 6400 km from the earth surface acceleration due to gravity is decreased by 75% of its value at the earth surface.

$\Rightarrow \quad h = R_e = 6400 \text{ km}$

(C) We know that the value of acceleration due to gravity, g is deduced to be:

$g = \frac{GM}{R^2}$ by the Law of Gravitational attraction. According to this, the value of, g depends on the mass and the radius of the Earth (G being a constant). However, as we know that the Earth is not perfectly spherical but bulged out (R is slightly more at the equator) at the equator on account of its rotation. Thus, it is found that the value of g is 0.5% more at the poles than it is at the equator. As a result, the weight ($W = mg$) of the same amount of sugar must also be greater at the poles.

Weight of 1 kg sugar at equator measured by the weighing machine = 1 × 9.8 = 9.8 N

Mass measured (as per the calibrated weighing machine) will be

$$= \frac{9.8}{9.8} = 1 \text{ kg}$$

weight of 1kg sugar at poles

$= 1 \times 9.85$

$= 9.85$ N.

Mass measured (as per the calibrated weighing machine) will be

$= \frac{9.85}{9.8}$

$= 1.005$ kg.

So at the poles, even 1 kg sugar will be read as 1.005 kg by the vendor and you will get lesser sugar from him. Thus, it is more profitable to buy sugar at the equator than on the poles. Mathematically it is,

$$g' = g - \omega^2 R_o \cos^2 \lambda$$

At poles, $\lambda = 90^o$, $\cos 90^o = 0$

At Equator, $\lambda = 0^o$, $\cos 0^o = 1$

$\therefore \quad g_{poles} > g_{equator}$

20. *To test the validity of the Copernicus model (Heliocentric model), the great Danish astronomer Tycho Braho made extraordinary observations by studying the motions of planets and stars without the aid of a telescope. This data was critically analyzed by Johannes Kepler. From these complicated data; Kepler deduced simple relations that governed planetary motion. These are three famous laws of Kepler which strongly supported the Copernicus model of the solar system and played a major role in the discovery of Newton's Law of Gravitation. The laws are (i) Law of orbits (ii) Law of areas (iii) Law of periods.* [Delhi Gov. QB 2022]

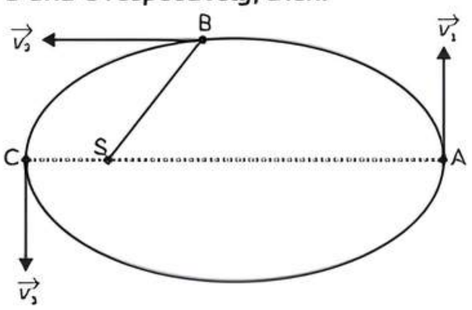
(A) Kepler's second law is the consequence of the law of conservation of:

(a) linear momentum
(b) energy
(c) angular momentum
(d) mass

(B) The distance of two planets from the sun is 10^{13} m and 10^{12} m respectively. The ratio of the periods is:

(a) $10\sqrt{10}:1$ (b) $1:10\sqrt{9}$
(c) $1:1$ (d) $10^3:1$

(C) The different positions of a planet around the sun in an elliptical orbit are shown by A, B and C. If v_1, v_2 and v_3 be the tangential speeds of the planet at A, B and C respectively, then:



(a) $v_1 = v_2 = v_3$ (b) $v_1 > v_2 > v_3$
(c) $v_1 < v_2 < v_3$ (d) $v_1 = v_2 > v_3$

(D) If the distance between the earth and the sun were one-third of its present value, the number of days in a year would have been:

(a) increased (b) decreased
(c) remains same (d) cannot say

(E) A planet moves around the sun in an elliptical orbit with the sun at one of its foci. The physical quantity associated with the motion of the planet that remains constant with time is:

(a) velocity
(b) centripetal force
(c) linear momentum
(d) angular momentum

Ans. (A) *(c) angular momentum*

Explanation: When a planet revolves around its orbit, it covers equal areas in similar amounts of time, according to Kepler's second rule of planetary motion, which states that $\frac{dA}{dt}$ = constant.

But, $\frac{dA}{dt} = \frac{L}{2m}$

Thus, L is constant.

(B) *(a) $10\sqrt{10}:1$*

Explanation: By Kepler's law $T^2 \propto r^3$

or $\quad \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

$\frac{T_1}{T_2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2}$

$\frac{T_1}{T_2} = 10\sqrt{10}$

(C) *(c) $v_1 < v_2 < v_3$*

Explanation: Kepler's Law

$\frac{dA}{dt}$ = constant and

$T^2 \propto r^3$

$\frac{dA}{dt}$ = constant

$v_1 < v_2 < v_3$

(D) *(b) decreased*

Explanation: According to Kepler's law

$T^2 \propto r^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$$\left(\frac{365}{T_2}\right)^2 = \left(\frac{3r_1}{r_1}\right)^3$$

$$\left(\frac{365}{T_2}\right)^2 = 27$$

$$\left(\frac{365}{T_2}\right) = 5.19$$

$$T_2 = \frac{365}{5.19}$$

$$T_2 = 70 \text{ days}$$

There will be 70 days in an year.

Hence, number of days will be decreased to approximately 70 days.

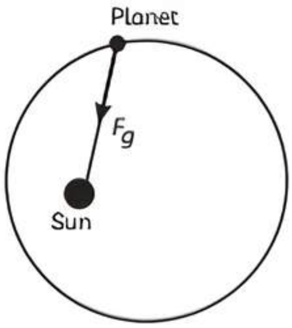
(E) *(d) angular momentum*

Explanation: A planet revolves around the sun, in an elliptical orbit under the effect of gravitational pull on the planet

So, torque, $T = |r \times F| = rF \sin 180^\circ = 0$

As $T = \frac{dL}{dt}$; so L = a constant

Hence, angular momentum is constant.



VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

21. Even though the moon is likewise a satellite of the earth, a person sitting in the satellite feels weightless, whereas a person standing on the moon feels weighted. Why? [Diksha]

Ans. In artificial satellites, there is no gravitation but on the surface of the moon gravity is present with its effect very little in amount. That is why a person feels weightlessness due to the absence of gravity on the artificial satellite and has some weight on the surface of the moon. Weightlessness is a phenomenon when the gravitational force is completely used up for providing centripetal acceleration.

22. The mass of the moon is nearly 10% of the mass of the earth. What will be the gravitational force of the earth on the moon, in comparison to the gravitational force of the moon on the Earth? [Delhi Gov. QB 2022]

Ans. Both forces will be equal in magnitude as gravitational force is a mutual force between the two bodies.

23. The tidal effect of the moon's pull is stronger than the tidal effect of the sun, despite the fact that the sun's force is greater. Explain.

Ans. Apart from the gravitational pull, the tidal effect also depends upon the cube of the distance between the two. Since, the distance between the earth and the sun is much larger than the distance between the sun and the moon so it not also balances but is more than the effect of gravitational force. Thus, the tidal effect of the moon's pull is greater than the tidal effect of the sun.

24. At what altitude above the earth's surface does the value of 'g' equal that of a 200-kilometer-deep mine?

Ans. Let the acceleration due to gravity at height h be the same as that at a depth d, deep into the earth *i.e.*, $d = 200$ km.

Therefore $$g' = g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$$\frac{2h}{R} = \frac{d}{R}$$

$$2h = d$$

$$h = \frac{d}{2} = 100 \text{ km}$$

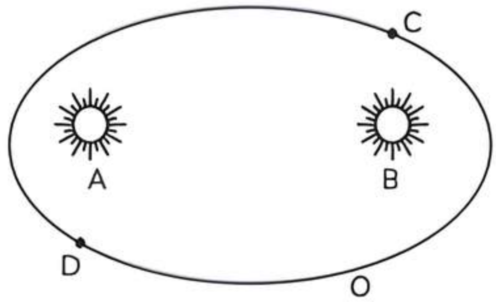
25. Molecules in air in the atmosphere are attracted by gravitational force of the earth. Explain why all of them do not fall into the earth just like an apple falling from a tree. [NCERT Exemplar]

Ans. Air molecules are random in motion due to thermal (temperature) energy. Air molecules and apples both are attracted by the earth by gravitational force but the resultant velocity of the air molecule is not exactly downward as apple as it has not any other motion except downward.

26. **When the finger is moved, the distance between the finger and the star changes hence the force of attraction between the finger and the star changes. This disturbs all the stars. How is this related to Newton's laws?**

Ans. According to Newton's law of gravitation, everybody in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. When we move our finger, the distance of the objects with respect to finger changes, disturbing the entire universe including stars.

27. **Identify the position of the sun in the following diagram if the linear speed of the planet is greater at C than at D.**



[Delhi Gov. QB 2022]

Ans. L = *mvr* constant

If *v* is maximum, *r* should be minimum

So, the Sun should be at B.

Hence, Sun should be at B as the speed of the planet is greater when it is closer to the Sun.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

28. **Suppose a spacecraft is going from earth to the moon and also planned to come back and it was filled with liquid propulsion during the calculation of filling the propulsion it was presumed that the maximum amount of fuel is burned during a journey from Earth to the Moon and back. Justify the reason.**

Ans. Newton's law of universal gravitation is usually stated that every particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. In the similar manner, when we go out from the earth's surface towards the surface of the moon, the force of gravitation acts as a resistance in not allowing the spaceship to move away from the surface of the earth and thus the spaceship has to put in an extra effort to move out of the surface of the earth because the gravitational pull exerted by the earth is more than that of the moon. Hence, fuel consumption will be more. On the other hand, when the spaceship goes back to the earth's surface from the surface of the moon, gravitational force will act in pulling the spaceship towards it, and thus the consumption of fuel will be less.

29. **If there is an attractive force between all objects, why do we not feel ourselves gravitating toward massive buildings in our vicinity? [Diksha]**

Ans. Gravity pulls us to massive buildings and everything else in the universe. Physicist Paul A.M. Dirac, winner of the 1933 Nobel Prize for physics, put it this way. "Pick a flower on earth and you move the farthest star" How much we are influenced by buildings or how much interaction there is between flowers and stars is another story. The forces between us and buildings are relatively small because their masses are small compared with the mass of the earth. The forces due to the stars are also small because of their great distances from us. These tiny forces escape our notice when they are overwhelmed by the overpowering attraction to earth.

30. **Earth's orbit is an ellipse with an eccentricity 0.0167. Thus, the earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant through the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain the variation of length of the day during the year?**

[NCERT Exemplar]

Ans. Angular momentum and areal velocity are constant as the earth orbits the sun.

At perigee, $r_p^2\omega_p^2 = r_a^2\omega_a^2$, at apogee

If '*a*' is the semi-major axis of the earth's orbit, then $r_p = a(1 - e)$ and $r_a = a(1 + e)$.

$$\frac{\omega_p}{\omega_a} = \left(\frac{1+e}{1-e}\right)^2.$$

$$e = 1.0667$$

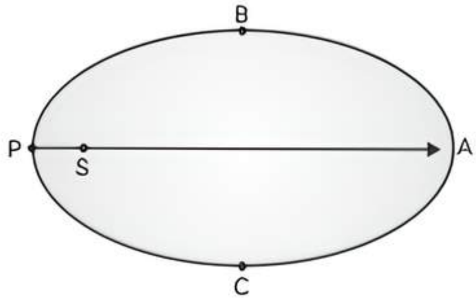
$$\frac{\omega_p}{\omega_a} = 1.0691$$

Let ω be angular speed which is the geometric the mean of ω_p and ω_a and corresponds to mean solar day.

$$\left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.0691$$

$$\left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.034$$

31. Let the speed of the planet at perihelion P in figure be v_p and the Sun planet distance SP be r_p. Relate (r_A, v_A) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal time to traverse BAC and CPB?



[Delhi Gov. QB 2022]

Ans. The magnitude of angular momentum at P is

$$L_p = m_p r_p v_p$$

Similarly, magnitude of angular momentum at

$$\text{A is } L_A = m_A r_A v_A$$

From the conservation of angular momentum

$$m_p r_p v_p = m_A r_A v_A$$

$$\frac{v_p}{v_A} = \frac{r_A}{r_p}$$

$$\because \quad r_A < r_p \therefore v_p > v_A$$

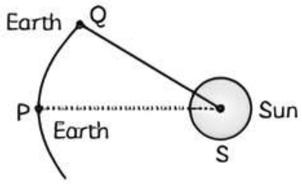
area bound by SB and SC (SBAC > SBPC)

By 2nd law, equal areas are swept in equal intervals of time. Time taken to transverse BAC > time taken to transverse CPB.

32. The mean solar day is the time interval between two successive noon when the sun passes through zenith point (meridian). Sidereal day is the time interval between two successive transit of a distant star through the zenith point (meridian). By drawing an appropriate diagram showing the earth's spin and orbital motion, show that mean solar day is four minutes longer than the sidereal day. In other words, distant stars would rise 4 minutes early every successive day.

[Hint: You may assume a circular orbit for the earth.] [NCERT Exemplar]

Ans.



Consider that on a day at noon, sun passes through the zenith(meridian). After one revolution (360°) of earth about its own axis sun again passes through the zenith.

During this time when the earth revolves at its own axis by 360° it changes its angle PSQ = 1°. So, 361° rotation by the earth is considered one solar day.

In 361° correspond to the = 24 hrs

1° will corresponds to $\frac{24}{361}$ hrs

$$= \frac{24}{361} \times 3600 \text{ sec}$$

= 3 min 59 sec = 4 min.

Hence, a distant star rises 4 min early every day.

33. If ω corresponds to 1° per day (mean angular speed), then p = 1.034 per day and a = 0.967 per day. Since 361° = 14 hrs: mean solar day, we get 361.034° which corresponds to 24 hrs 8.14'' (8.1'' longer) and 360.967° corresponds to 23 hrs 59 min 52'' (7.9'' smaller). This does not explain the actual variation of the length of the day during the year. Whether the weight of the body should be greater at night when the earth and sun are attracted in the same direction, or during the day when the sun and earth are attracted in opposite directions.

Ans. No, the earth is a satellite of the sun. Everybody placed on earth is also a satellite of the sun. Both the body and earth will have the same acceleration towards the sun. Hence, there will be no relative gravitational acceleration between the body and the earth. It means a body placed on earth will not experience any gravitational effect due to the sun. It will experience a gravitational force of attraction due to the earth, which will be the weight of the body measured on the surface of the earth, which will remain the same for all 24 hours.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

34. **Maria and Shaina are good friends and living in a city near equator. Maria went to a country located near the north pole of the earth with her parents. Her friend Shaina requested her to bring a gold necklace as gold was cheaper in that country. Maria purchased the necklace weighing 20 g and handed it over to Shaina. When Shaina got the necklace weighed by the local goldsmith, its weight was less than 20 g. Shaina told Maria that she was cheating her. However, Maria explained that the weight of the body at the equator is less than at poles. Maria asked Shaina to return the necklaces to her because she was not interested in spoiling her friendship. Why is the weight of the necklace less at the equator than at the poles?**

Ans. Weight of object = mg.

If g is less than the weight. The reasons are:

(1) The shape of the earth is an oblate spheroid. That's why the radius at the equator is more. Thus, g is less at the equator.

(2) The centrifugal force $\left(\frac{mv^2}{R}\right)$ is opposite to g at the equator (the angle between them being 180°). Since forces are vectors and the resultant vector is minimum when the vectors are opposite to each other, g is minimum at the equator. If the earth rotated extremely fast then the centrifugal force and g would have cancelled each other out at the equator.

35. **If a planet existed where mass and radius were both halves those of the earth, what would be the value of the acceleration due to gravity on its surface as compared to what it is on the earth's surface?**

Ans. $$g = \frac{GM}{R^2}, g_o = \frac{GM_o}{{R_o}^2}$$

where $$M' = \frac{1}{2}M \quad \text{and } R' = \frac{1}{2}R$$

$$\frac{g'}{g} = \frac{1}{2} \times 2^2 = 2$$

$$g' = 2g$$

36. **The mass and diameter of a planet are twice that of the earth. What will be the period of oscillation of a simple pendulum on this planet, if it is a second's pendulum on the earth?**

Ans. $g = \frac{GM}{R^2}, g_e = \frac{GM_e}{R_p^2}$, and $g_p = \frac{GM_p}{R_p^2}$

Here $M_p = 2M_e$ and $R_p = 2R_e$

$$\frac{g_p}{g_e} = \frac{1}{2}$$

Time period of a simple pendulum

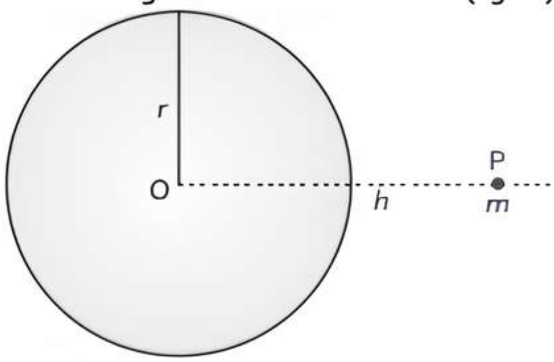
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}}$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$T_p = \sqrt{2}T_e = \sqrt{2} \times 2 = 2\sqrt{2} sec$$

37. **A mass m is placed at P a distance h along the normal through the centre O of a thin circular ring of mass M and radius r (figure).**



If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$?

[NCERT Exemplar]

Ans. A ring of radius r of mass M and a mass m is placed at a distance h axially at P. Then the gravitational force F at P

$$F_h = \frac{GMm\cos\theta}{AP^2} = \frac{GMmh}{(r^2+h^2)^{3/2}}$$

$$\left[\therefore \cos\theta = \frac{h}{(r^2+h^2)^{1/2}}\right]$$

$$\frac{F_r}{F_{2r}} = \frac{\frac{GMmr}{(r^2+h^2)^{3/2}}}{\frac{GMm2r}{(r^2+h^2)^{3/2}}}$$

$$= \frac{1}{(2r^2)^{3/2}} \times \frac{(r^2+4r^2)^{3/2}}{2}$$

$$\frac{F_r}{F_{2r}} = \frac{1}{2\sqrt{2}r^3} \times \frac{(5r^2)^{3/2}}{2}$$

$$= \frac{5\sqrt{5}r^3}{4\sqrt{2}r^3}$$

$$\frac{F_r}{F_{2r}} = \frac{5}{4}\sqrt{\frac{5}{2}}$$

Or $$\frac{F_{2r}}{F_r} = \frac{4}{5}\sqrt{\frac{2}{5}}$$

Gravitational force on m at a distance $2r$ from O is the $\frac{4}{5}\sqrt{\frac{2}{5}}$ time the gravitational force when m is placed at r distance from O.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

38. Consider an apparatus consists of two pairs of spheres, each pair forming dumbbells that have a common swivel axis. One dumbbell is suspended from a quartz fiber and is free to rotate by twisting the fiber; the amount of twist is measured by the position of a reflected light spot from a mirror attached to the fiber. The second dumbbell can be swivelled so that each of its spheres is in close proximity to one of the spheres of the other dumbbell; the gravitational attraction between two sets of spheres twists the fiber, for what purpose these twists are used.

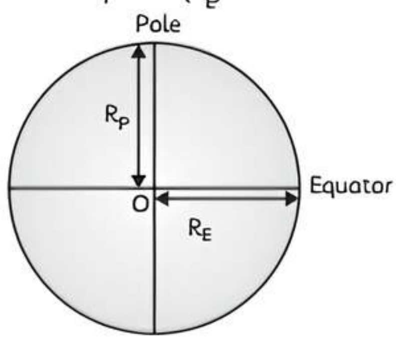
Ans. It is the measure of this twist that allows the magnitude of the gravitational force to be calculated. And used by Cavendish. Cavendish experiment is used for measuring the gravitational force of attraction between pairs of lead spheres, which allows the calculation of the value of the gravitational constant, G. In Newton's law of universal gravitation, the attractive force between two objects (F) is equal to G times the product of their masses (m_1m_2) divided by the square of the distance between them (r^2);

i.e., $$F = G\frac{m_1m_2}{r^2}.$$

The experiment was performed in 1797–98 by the English scientist Henry Cavendish. The apparatus featured a torsion balance, a wooden rod was suspended freely from a thin wire, and a lead sphere weighing 0.73 kg (1.6 pounds) hung from each end of the rod. A much larger sphere, weighing 158 kg (348 pounds), was placed at each end of the torsion balance. The gravitational attraction between each larger weight and each smaller one drew the ends of the rod aside along a graduated scale. The attraction between these pairs of weights was counteracted by the restoring force from a twist in the wire, which caused the rod to move from side to side like a horizontal pendulum. The Cavendish experiment was significant not only for measuring Earth's density (and thus its mass) but also for proving that Newton's law of gravitation worked on scales much smaller than those of the solar system.

39. Discuss how the shape of the earth influences the variation of *g* on its surface. When a body is taken from the equator to the pole, why does its weight increase?

Ans. The earth is not perfectly spherical but is an oblate spheroid. The polar radius (radius near the poles) earth's is 21 km smaller than its equatorial radius (near the equator). Earth is not spherical, but actually, it is bulged out as per the formula derived, the acceleration due to gravity is inversely proportional to the square of the radius of the earth. The radius of the earth at the equator is more; at the equator, g is less. This is the reverse in the case of poles. Suppose we consider the shape of the earth as slightly elliptical. So, we will have different distances from the pole and the equator from the center. So, the distance between the pole (R_P) and the equator (R_E) from the center is as,



$R_E > R_P$

And, from observation, we have the relation between R_E and R_P as,

$$g \propto \frac{1}{R^2}$$

Then, if we consider G and M as constants in the acceleration formula, then,

$$g_P \propto \frac{1}{R_p^2} \text{ and}$$

$$g_E \propto \frac{1}{R_E^2}$$

So, the gravitational accelerations on the equator and pole are given by,

$$g_P > g_E$$

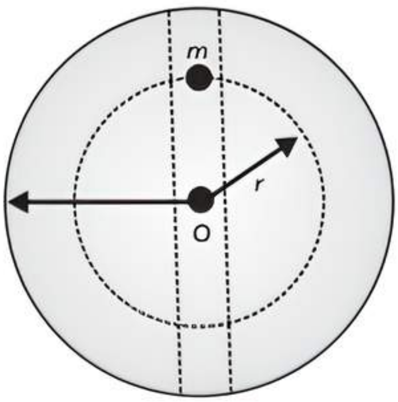
Here, from the distances between the poles and the equator, we have the relationship of gravitational acceleration as,

$$\frac{g_p}{g_e} = \frac{R_E^2}{R_P^2}$$

Hence, the gravitational acceleration at the equator is less than the gravitational acceleration at the pole.

40. We have a huge sphere of mass M, radius R and of uniform density ρ. A body of mass *m* at a distance *r* > R from the center of the sphere experiences a gravitational force which varies as $\frac{1}{r^2}$. Now a hole is drilled along the diameter of the sphere. The gravitational force experienced by the body of mass m at a distance *r* < R varies as *r*. Explain.

Ans. When the body is at a distance $r < R$ from the center of the sphere, then it is inside a spherical shell of radius R and at the surface of a solid sphere of radius *r*. The body doesn't experience gravitational force inside the spherical shell. The only gravitational force experienced by the body is due to mass '*M*' of the solid sphere of radius '*r*'.



Therefore, $F = G\frac{M'm}{r^2}$.

Now, M' = mass per unit volume of a sphere of radius R × volume of a sphere of radius *r*.

$$F = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi R^3 = \frac{Mr^3}{R^3} \quad \text{–(i)}$$

Putting this value in equation (i).

$$F = \frac{GMm}{R^3} r \text{ or } F \propto r$$

41. A star like a sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let *r* be the distance of the body from the centre of the star and let its linear velocity be *v*, angular velocity ω, kinetic energy K, gravitational potential energy U, total energy E and angular momentum L. As the radius *r* of the orbit increases, determine which of the above quantities increase and which ones decrease. [NCERT Exemplar]

Ans. Let us consider a body of mass *m* rotating around the star *S* of mass *m* in a circular path of radius r.

(1) Then the orbital velocity

$$v_o = \frac{\sqrt{GM}}{r}$$

or $$v_o \propto \frac{1}{\sqrt{r}}$$

Hence, on increasing the radius of the circular path orbital velocity decreases.

(2) Angular velocity, $\omega = \frac{2\pi}{T}$

and $T_2 \propto r^3$ by Kepler's third law

$$\omega = \frac{2\pi}{Kr^{3/2}}$$

or $$\omega \propto \frac{1}{\sqrt{r^3}}$$

Hence, on increasing the radius of the circular orbit the angular velocity decreased.

(3) Kinetic energy, $E_k = \frac{1}{2}m\frac{GM}{r}$ or $E_k \propto \frac{1}{r}$.

Hence on increasing the radius of the circular path the kinetic energy decreased.

(4) Gravitation potential energy, $E_p = \frac{-GMm}{r}$

$$\text{or } E_p \propto \left(\frac{1}{r}\right)$$

So, on increasing radius of circular orbit the P.E. (E_p) increases.

(5) Total energy

$$E = E_k + E_p = \frac{GMm}{2r} + \left(\frac{-GMm}{r}\right)$$

$$E = \frac{GMm}{2r}$$

Hence, on increasing the radius of the circular orbit the total energy E will also be increased.

(6) Angular momentum, $L = mvr = \sqrt{\frac{GM}{r}}r$

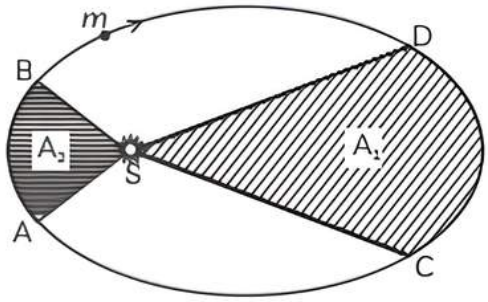
$$L = m\sqrt{GM}$$

or $$L \propto \sqrt{r}$$

Hence, the increasing radius r of the circular orbit increases the angular momentum.

NUMERICAL Type Questions

42. **The figure shows the elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the shaded area of SAB. If t_1 is the time for the planet to move from D to C and t_2, is the time to move from A to B, what is the relation between t_1 and t_2?**



[Delhi Gov. QB 2022](2m)

Ans. According to Kepler's law of areas:

The line that joins any planet to the sun sweeps out equal areas in equal intervals of time

i.e., $\frac{\Delta A}{\Delta t}$ is constant.

Given that:

Area SCD = 2 × Area SAB

Using Kepler's law

$$\frac{\Delta A_{SCD}}{\Delta t_{SAB}} = \frac{t_1}{t_2} = 2$$

43. **At what depth below the surface does the acceleration due to gravity becomes 70% of its value on the surface of the earth's? (2m)**

Ans. $$G_{depth} = g_{surface}\left(1 - \frac{d}{R_e}\right)$$

$$\Rightarrow \frac{7}{10}g = g\left(1 - \frac{d}{R_e}\right)$$

$$\Rightarrow d = \frac{3}{10} \times 6400 \text{ km}$$

$$= 1920 \text{ km}$$

44. **The mass of the planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30} kg. The mean distance of Jupiter from the Sun is 7.8×10^{11} m. Calculate the gravitational force which the sun exerts on Jupiter, and the speed of Jupiter. [Delhi Gov. QB 2022](2m)**

Ans. We know that,

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{(7.8 \times 10^{11})}$$

$$F = 4.1 \times 10^{23} \text{N}$$

$$\therefore V = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{GMm}{r^2} \times \frac{r}{m}}$$

$$V = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1.9 \times 10^{30}}{7.8 \times 10^{11}}}$$

$$V = 1.3 \times 10^4 \text{ ms}^{-1}$$

45. **What is the value of acceleration due to gravity at a height equal to half the radius of the earth, from the surface of the earth? (Take g = 10 m/s² on earth's surface). (2m)**

Ans. $$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

$$\left[\text{Given: } h = \frac{R_e}{2}\right]$$

$$g' = \frac{g}{1 + \left(\frac{R_e}{2R_e}\right)^2}$$

$$g' = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{g}{\left(\frac{3}{2}\right)^2} = \frac{4g}{9}$$

$$\therefore h = \frac{R_e}{2}$$

$$\Rightarrow g' = \frac{4g}{9} = 4.44 \text{ m/s}^2$$

46. **A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket zero? Mass of sun = 2×10^{30} kg.**

Mass of the earth = 6×10^{24} kg.

Neglect the effect of other planets etc. Orbital radius = 1.5×10^{11} m.

[Delhi Gov. QB 2022](3m)

Ans. Given, Mass of the sun, $M_s = 2 \times 10^{30}$ kg

Mass of the Earth, $M_e = 6 \times 10^{24}$ kg

Orbital radius, $r = 1.5 \times 10^{11}$ kg

Mass of the rocket = m

Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton's law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = \frac{GmM_e}{x^2}$$

$$\left[\frac{(r-x)^2}{x}\right]^2 = \frac{M_s}{M_e}$$

$$\frac{(r-x)}{x} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$1.5 \times 10^{11} - x = 577.35x$$

$$578.35x = 1.5 \times 10^{11}$$

$$x = \frac{1.5\times10^{11}}{578.35}$$

$$= 2.59 \times 10^{8}\ \text{m}$$

47. At what height from the surface of the earth will the value of 'g' be reduced by 36% of its value at the surface of Earth?

[Delhi Gov. QB 2022] (3m)

Ans. Given,

Radius of earth, $R = 6400$ km

Gravity reduced 36%, $g' = \left(1-\frac{36}{100}\right)g = 0.64\ g$

Gravity acceleration above height h from the earth's surface.

$$g = g\left[\frac{R}{R+h}\right]^2$$

$$0.64g = g\left[\frac{R}{R+h}\right]^2$$

$$0.8(R+h) = R$$

$$h = \frac{R}{4} = \frac{6400}{4} = 1600\ \text{km}$$

Hence, height is one-fourth of the earth radius, 1600 km.

48. The acceleration due to gravity at the Moon's surface is 1.67 ms^{-2}. If the radius of the Moon is 1.74×10^6m. Calculate the mass of the Moon. [Delhi Gov. SQP 2022](3m)

Ans. Given: Acceleration due to gravity of moon

$= 1.67\ ms^{-2}$

radius of moon = 1.74×10^6m

To find: Mass of the moon

We know: $g = \frac{GM}{R^2}$

$$\frac{gR^2}{G} = M \quad \text{...(i)}$$

Substituting given values in eqn. (i)

$$\frac{1.67\times(1.74\times10^6)^2}{6.67\times10^{-11}} = M$$

$$7.58 \times 10^{22} = M$$

$\therefore$ Mass of moon is 7.58×10^{22} kg

GRAVITATIONAL ENERGY AND SATELLITE MOTION 2

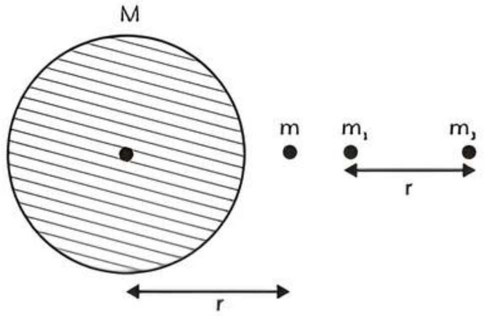
TOPIC 1

GRAVITATIONAL POTENTIAL ENERGY

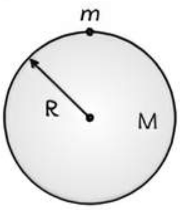
The gravitational potential energy of a particle situated at a point in some gravitational field is defined as the amount of work required to bring it from infinity to that point without changing its kinetic energy.

$$W = U = -\frac{Gmm}{r} \text{ or } U = \frac{Gm_1m_2}{r}$$

(Here negative sign shows the boundedness of the two bodies)



It is a scalar quantity. The SI unit of Gravitational Potential energy is joule and Dimension are $[M^1L^2T^{-2}]$.

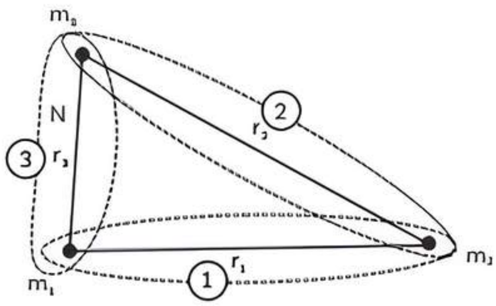


The gravitational potential energy of a particle of mass '*m*' placed on the surface of the earth of mass 'M' and radius 'R' is given by:

$$U = -\frac{GMm}{R}$$

Gravitational Potential Energy For Three Particle System

If there are more than two particles in a system, then the net gravitational potential energy of the whole system is the sum of the gravitational potential energies of all the possible pairs in that system.

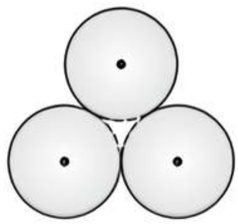


$$U_{system} = \left(-\frac{Gm_1m_2}{r_1}\right) + \left(-\frac{Gm_2m_3}{r_2}\right) + \left(-\frac{Gm_1m^3}{r_3}\right)$$

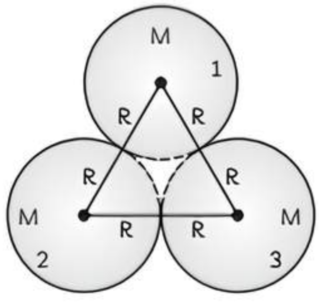
$$U_{system} = -\frac{Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_3m_1}{r_3}$$

If the earth stops rotating about its axis, then the apparent weight of bodies or effective acceleration due to gravity will increase at all the places except poles.

Example 2.1: Three solid spheres of mass M and radius R are placed in contact as shown in figure. Find the potential energy of the system.



Ans. $PE = PE_{12} + PE_{23} + PE_{31}$



$$= -\frac{GM^2}{2R} - \frac{GM^2}{2R} - \frac{GM^2}{2R}$$

$$PE = -\frac{3GM^2}{2R}$$

TOPIC 2

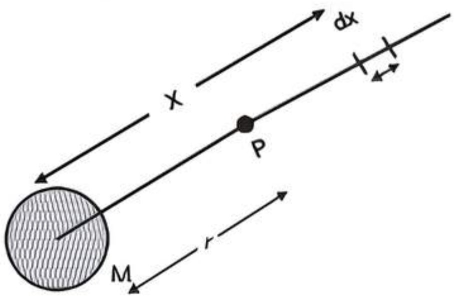
GRAVITATIONAL POTENTIAL

The gravitational field around a material body can be described not only by gravitational intensity $\vec{I_g}$, but also by a scalar function, the gravitational potential V. The gravitational potential is the amount of work done by an external agent in bringing a body of unit mass from infinity to that point without changing its kinetic energy. $V = \frac{W_{ext}}{m}$

Gravitational force on unit mass at (P) will be

$$F = \frac{Gm(1)}{x^2} = \frac{GM}{x^2}$$

Work done by this force when the unit mass is displaced through the distance dx is:



$$dW_{ext} = Fdx = \int_{\infty}^{r} \frac{GM}{x^2} = -\left(\frac{GM}{x}\right)_{\infty}^{r} = -\frac{Gm}{r}.$$

This work done is the measure of gravitational potential at point (P)

$$\therefore \quad V_p = -\frac{GM}{r}$$

If $r = \infty$ then $V_{\infty} = 0$.

Hence, gravitational potential is maximum at infinity(as it is a negative quantity at a point P).

Then, If $r = R_e$ (on the surface of the earth).

$$\therefore \quad V_e = -\frac{GM_e}{R_e}$$

Relation between intensity and potential gradient

$$V = -\int \vec{I}.\vec{dr}$$

$$\Rightarrow \quad dV = -\vec{I}.\vec{dr}$$

$$\therefore \quad I = -\frac{dV}{dr}$$

= -ve potential gradient.

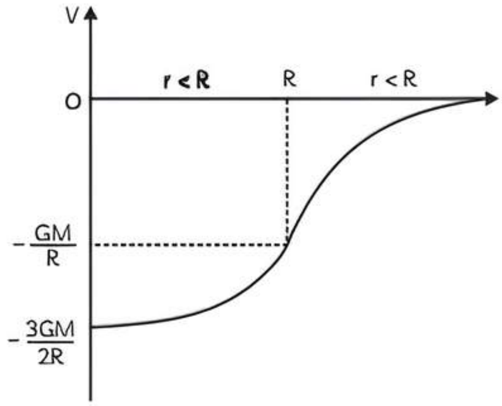
Gravitational Potential Due to Solid Sphere and Spherical Shell

Solid sphere

Case 1: $r > R$ (outside the sphere): $V_{out} = -\frac{GM}{r}$

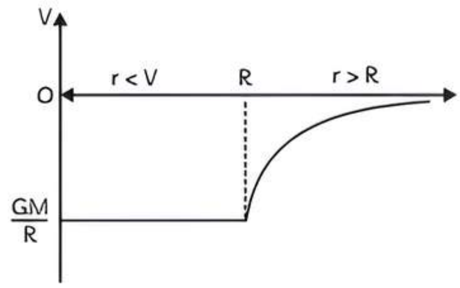
Case 2: $r = R$ (on the surface): $V_{surface} = -\frac{GM}{R}$

Case 3: $r < R$ (Inside the sphere): $V_{in} = -\frac{GM}{2R^2}[3R^2 - r^2]$



It is clear that the potential V will be minimum at the centre ($r = 0$) but maximum in magnitude.

Spherical shell



Case 1: $r > R$ (outside the sphere): $V_{out} = -\frac{GM}{r}$

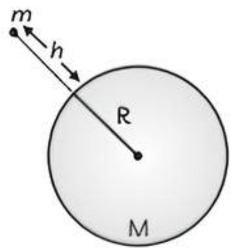
Case 2: $r = R$ (on the surface): $V_{surface} = -\frac{GM}{r}$

Case 3: $r < R$ (inside the sphere): $V_{in} = -\frac{GM}{R}$

Potential is same everywhere and is equal to its values at the surface.

Example 2.2: A projectile of mass m is thrown vertically up with an initial velocity v from the surface of the earth (mass of earth = M). If it comes to rest at a height h. What is the change in its potential energy?

Ans. Given:



$$U_1 = -\frac{GMm}{R}$$

$$U_2 = -\frac{GMm}{R+h}$$

$$\Delta U = U_2 - U_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$= GmM\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GmMh}{R(R+h)}$$

TOPIC 3

ESCAPE VELOCITY AND ESCAPE ENERGY

Escape Velocity

It is the minimum velocity required for an object located at the planet's surface so that it just escapes the planet's gravitational field.

Consider a projectile of mass m, leaving the surface of a planet (for some other astronomical body or system), when the projectile just escapes to infinity, it has neither kinetic energy nor potential energy.

From the conversion of mechanical energy:

$$\frac{1}{2}mv_e^2 + \left(-\frac{GMm}{R}\right) = 0 + 0$$

$$\Rightarrow \qquad v_e = \sqrt{\frac{2GM}{R}}$$

The escape velocity of a body from a location which is at height 'h' above the surface of a planet.

we can use:

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}} \qquad (\because r = R + h)$$

Where, r = Distance from the center of the planet

H = height above the surface of the planet.

Escape speed depends on:

(1) Mass (M) and radius(R) of the planet.

(2) Position from where the particle is projected

Escape speed does not depend on:

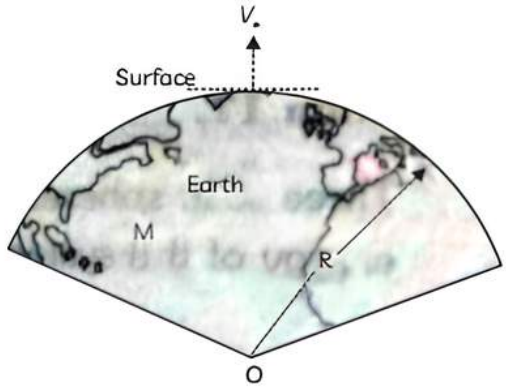
(1) Mass (m) of the body which is projected.

(2) Angle of projection

If a body is thrown from the earth's surface with escape speed, it goes out of earth's gravitational field and never returns to the earth's surface.

Escape Energy

Minimum energy is given to a particle in the form of kinetic energy so that it can just escape the earth's gravitational field.



Magnitude of escape energy $= \frac{GMm}{R}$

(-ve of PE on the earth's surface)

Escape energy = kinetic energy corresponding to the escape velocity

$$\Rightarrow \qquad \frac{GMm}{R} = \frac{1}{2}mv_e^2$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

Example 2.3: If the velocity given to an object from the surface of the earth is n times the escape velocity, then what will be its residual velocity at infinity? [NCERT]

Ans. Let the residual velocity be v, then from energy conversion

$$\frac{1}{2}m(nv_e)^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow \qquad v^2 = n^2v_e^2 - \frac{2GM}{R}$$

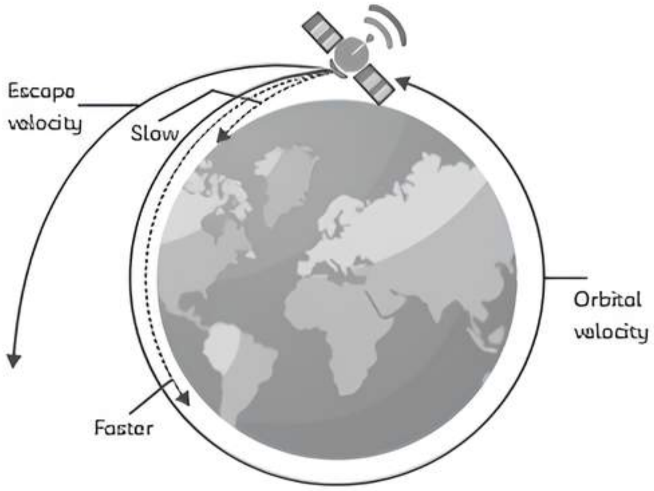
$$= n^2v_e^2 - v_e^2$$

$$= (n^2 - 1)v_e^2$$

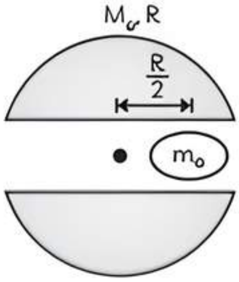
$$= \left(\sqrt{n^2 - 1}\right)v_e$$

Example 2.4 Case Based:

In astronomy and space research, escape velocity is the velocity at which a body can escape from a gravitational center of attraction without further acceleration. The velocity required to maintain a circular orbit at the same altitude is equal to the square root of 2 (or around 1.414) times the escape velocity. If atmospheric resistance were ignored, the escape velocity at the Earth's surface would be around 11.2 kilometers (6.96 miles) per second. At its surface, the less massive Moon's escape velocity is around 2.4 km per second. If the planet's escape velocity is low enough to be near the average velocity of the gas, the planet (or satellite) cannot keep an atmosphere for long.



(A) A narrow tunnel is dug along the diameter of the earth, and a particle of mass m_0 is placed at $\frac{R}{2}$ distance from the center. Find the escape speed of the particle from that place.



(B) The escape velocity for a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction, and passes through a smooth tunnel through its center. Its speed at the center of the planet will be:

(a) $\sqrt{1.5}\,v_e$ (b) $\frac{v_e}{\sqrt{2}}$

(c) v_e (d) Zero

(C) A mass of 6×10^{24} kg(mass of the earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is 3×10^8 m/s(equal to the velocity of light). What should be the radius of the sphere?

(D) With what velocity must a body be thrown from the earth's surface so that it may reach a height $4R_e$ above the earth's surface? (radius of the earth, R_e = 6400 km, g = 9.8 m/s^2).

(E) Assertion (A): The escape velocity from the surface of Jupiter is found to be less than that from the earth's surface.

Reason (R): The radius of Jupiter is smaller than that of earth.

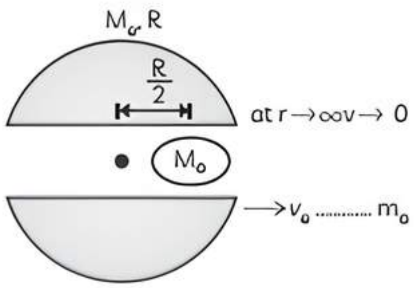
(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

Ans. (A) Suppose we project the particle with speed v_e, so that is just reaches infinity ($r \to \infty$). Applying the energy conservation principle.



$$K_1 + U_1 = 0$$

$$\frac{1}{2}m_0 v_e^2 + m_0\left[-\frac{GM_e}{2R^2}\left\{3R^2 - \left(\frac{R}{2}\right)^2\right\}\right] = 0$$

$$\Rightarrow \quad v_e = \sqrt{\frac{11GM_e}{4R}}$$

(B) (a) $\sqrt{1.5}\,v_e$

Explanation: From mechanical energy conservation.

$$0 + 0 = \frac{1}{2}mv^2 - \frac{3GMm}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{3GM}{R}} = \sqrt{1.5}\,v_e.$$

(C) The mass of the sphere = 6×10^{24} kg. Escape velocity = 3×10^8 m/s.

As $v_e = \sqrt{\frac{2GM}{R}}$, $R = \frac{2GM}{v_e^2}$

$$R = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2}$$

$= 9 \times 10^{-3}$ m = 9 mm.

(D) By using conservation of mechanical energy

$$=\frac{1}{2}m_0v^2-\frac{GMm_0}{R_e}$$

$$=0-\frac{GMm_0}{(R_e+4R_e)}$$

$$=\frac{1}{2}m_0v^2=-\frac{GMm_0}{5R_e}+\frac{GMm_0}{R_e}\Rightarrow\frac{1}{2}m_0v^2$$

$$=\frac{4}{5}\frac{GMm_0}{R_e}$$

$$\Rightarrow\quad v^2=\frac{8}{5}\frac{GM}{R_e}=\frac{8}{5}\frac{gG^2_e}{R_e}$$

$$v^2=\frac{8}{5}\times 9.8\times 6400\times 10^3=10^3$$

$$\Rightarrow\quad v=10^4\text{ m/s}$$

(E) *(d) A is false and R is also false.*

Explanation: The escape velocity on the earths surface is given by the equation,

$$v=\sqrt{\frac{GM}{R}}$$

Where, G be the Gravitational constant M be the mass of the earth and R be the radius of the earth.

As we all know that the radius of Jupiter is found to be 11 times greater than that of the earth. There is a difference in acceleration due to gravity also. The acceleration due to gravity on Jupiter surface is 2.36 times the acceleration due to gravity of the earth's surface. Therefore, the escape velocity from Jupiter is surely higher than that from the surface of earth.

TOPIC 4

SATELLITE MOTION

A light body revolving around a heavier planet due to gravitational attraction is called a satellite. The moon is the natural satellite of the earth.

(1) Natural satellite (e.g. Moon revolving around earth)

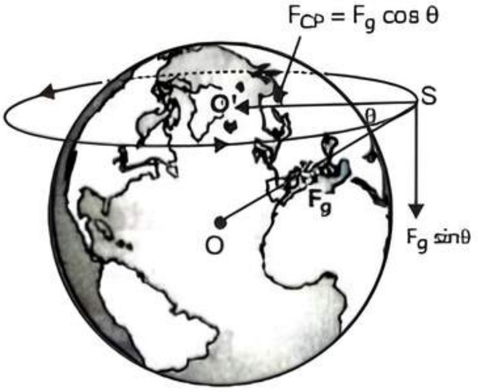
(2) Artificial satellite

(3) Polar satellite

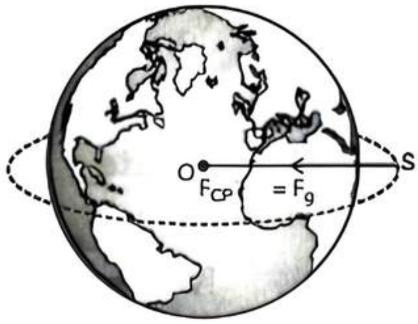
Essential Conditions for Satellite Motion

(1) The center of the satellite's orbit should coincide with the center of the earth.

(2) The plane of the orbit of the satellite should pass through the center of earth.



(A) Unstable orbit
(Due to $F_g\sin\theta$, orbit will shift)



(B) Stable orbit

It follows that a satellite can revolve around the earth only in those circular orbits whose centers coincide with the center of the earth. Circles drawn on the globe with centers coincident with the earth are known as great circles. Therefore, a satellite revolves around the earth along circles concentric with great circles.

Orbital Velocity

A satellite of mass m moving in an orbit of radius r with speed v_0, the required centripetal force is provided by gravitation.

$$F_{cp}=F_g$$

$$\frac{mv_0^2}{r}=\frac{GMm}{r_2}$$

$$v_0=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{(R_e+h)}}\qquad(r=R_e+h)$$

For a satellite very close to the earth's surface,

$$h << R_e$$
$$R = R_e$$
$$v_0 = \frac{GM}{R_e}$$
$$= \sqrt{gR_e} = 8 \text{ km/s}$$

If a body is taken to some height (small) from the earth and given a horizontal velocity of magnitude 8 km/s then it becomes a satellite of the earth.

v_0 depends upon the mass of the planet, radius of the circular orbit of the satellite. If the orbital velocity of a satellite becomes $\sqrt{2}v_0$ (or increased by 41.4%) or K.E. is doubled then it escapes from the gravitational field of the earth.

Energy Period of a Satellite

$$T = \frac{2\pi r}{v_0} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}}$$

$$= \frac{2\pi r^{\frac{3}{2}}}{R\sqrt{g}}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2}{GM} r^3$$

$$\Rightarrow \quad T^2 \propto r^3 \qquad (r = R + h)$$

For a satellite close to the earth's surface,

$$v_0 = \sqrt{\frac{GM_e}{R_e}} = 8 \text{ km/s}$$

$$T_0 = 2\pi\sqrt{\frac{R_e}{g}}$$

= 84 minutes
= 1 hour 24 minutes
= 1.24 = 5040 s

In terms of density, $T_0 = \frac{2\pi (R_e)^{\frac{3}{2}}}{\left(G \times \frac{4}{3}\pi R_e \times \rho\right)^{\frac{1}{2}}} = \sqrt{\frac{3\pi}{G\rho}}$.

Time period of a nearby satellite only depends on the density of the planet.

For Moon:
h_m = 3800,00 km and T_m = 27 days

$$v_o = \frac{2\pi(R_e + h)}{T_m} = \frac{2\pi(386400 \times 10^3)}{27 \times 24 \times 60 \times 60} = 1.04 \text{ km/s}$$

Energy of a satellite:
Kinetic energy K.E.

$$= \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$$

$$= \frac{L^2}{2mr^2} (L = mrv_0 = m\sqrt{GMr})$$

Potential energy, P.E. $= \frac{GMm}{r} = -mv_0^2 = -\frac{L^2}{mr^2}$

Total mechanical energy,
T.E = P.E + .K.E

$$= -\frac{mv_0^2}{2} = -\frac{GMm}{2r}$$

$$= -\frac{L^2}{2mr^2}$$

Binding energy: Total mechanical energy (potential + kinetic) of a closed system is negative. The modules of this total mechanical energy are known as the binding energy of the system. This is energy due to which system is bound or the different parts of the system are bonded to each other.

Binding energy of a satellite (system):

B.E = – T.E

$$B.E = \frac{1}{2}mv_0^2$$

$$= \frac{L^2}{2mr^2}.$$

Hence, $\quad B.E = K.E = -T.E = \frac{-P.E}{2}$

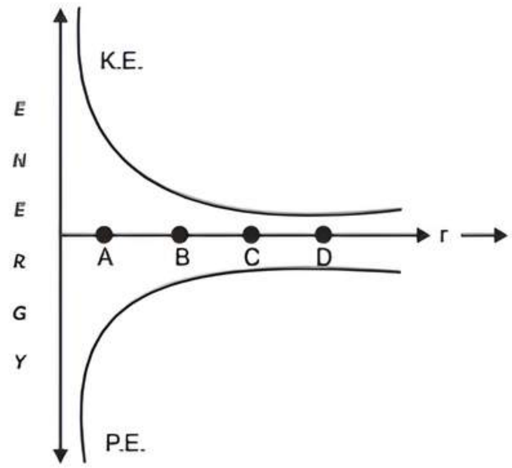
Escape energy and ionisation energy are practical examples of binding energy.

Work done in changing the orbit of a satellite:
W = change in mechanical energy of the system

but $\quad E = \frac{-GMm}{2r}$

So $\quad W = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

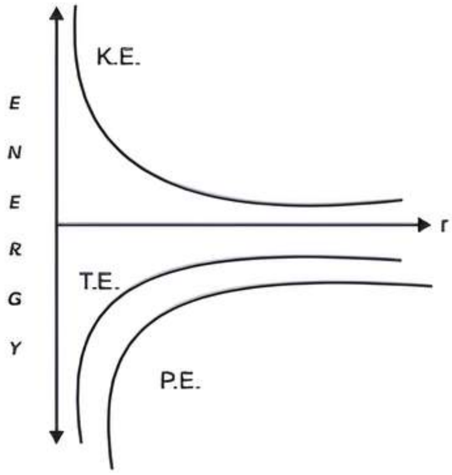
In the given graph of energy vs position
At point A:



$\therefore$ | (P.E.) > K.E. $\therefore$ KE + PE = −ve|

At A, B and C system is bounded.

At point D:



$\therefore$ |PE| > |K.E.|

$\therefore$ TE = KE + PE = 0

So, the system is unbounded.

$$K.E. = \frac{GMm}{3r}$$

$$P.E. = \frac{GMm}{r}$$

$$T.E. = \frac{GMm}{2r}$$

Example 2.5: A satellite launching station should be:

(a) near the equatorial region

(b) near the polar region

(c) on the polar axis

(d) all locations are equally good [NCERT]

Ans. *(a) near the equatorial region*

Explanation: Near the equator, '*g*' is minimum due to the rotation and shape of the earth which help to minimize fuel consumption.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. In a circular orbit near the earth's surface, a space shuttle is launched. The additional velocity that will be delivered to the space shuttle in order to release it from the gravitational pull will be:

(a) 1.52 km/s (b) 2.75 km/s

(c) 3.28 km/s (d) 5.18 km/s

Ans. *(c) 3.28 km/s*

Explanation: For escape from earth's gravitational field T.E. must be zero.

From conservation of energy

KE + PE = 0

$$v_0 = \sqrt{\frac{GM}{R}}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

additionally, velocity, $= v_e - v_0$

$= 11.2 - 8 = 3.2$ km/s

2. If the length of the day is T, the height of that TV satellite above the earth's surface which always appears stationary from the earth, will be:

(a) $h = \left[\frac{4\pi^2 GM}{T^2}\right]^{\frac{1}{3}}$ (b) $h = \left[\frac{4\pi^2 GM}{T^2}\right]^{\frac{1}{2}}$

(c) $h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{3}} - R$ (d) $h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{2}} + R$

Ans. *(c)* $h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{3}} - R$

Explanation:

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$\Rightarrow \quad h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{3}} - R$$

3. Energy required to move a satellite of mass m from an orbit of radius 2R to 3R is, (M mass of earth):

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$

(c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

[Delhi Gov. QB 2022]

Ans. *(d)* $\frac{GMm}{6R}$

Explanation: Energy required = (Final potential-Initial potential) *m*

$$\text{Energy required} = -\frac{GMm}{3R} - \frac{-GMm}{2R} = \frac{GMm}{6R}$$

4. The gravitational force between two bodies is directly proportional to $\frac{1}{R}$ (not $\frac{1}{R^2}$), where 'R' is the distance between the bodies. Then the orbital speed for this force in a circular orbit is proportional to:

(a) $\frac{1}{R^2}$

(b) R^0 (independent of R)

(c) R

(d) $\frac{1}{R}$

Ans. *(b) R^0 (independent of R)*

Explanation: Centripetal force is equal to gravitational force and given as.

$$F_{CP} = F_g$$

$$\frac{mv^2}{R} = \frac{GM_e m}{R}$$

$$\Rightarrow \quad v^2 = \frac{GM_e}{R^0}$$

5. A planet revolves around the sun; its apogee distance from the sun is r_A and that at perigee is r_p. The masses of the planet and the sun are m and M, respectively. The velocity of the planet at apogee and perigee, respectively, v_A and v_p and T is the time period of revolution of the planet around the sun.

(a) $T^2 = \frac{\pi^2}{2Gm}(r_A + r_p)^2$

(b) $T^2 = \frac{\pi^2}{2GM}(r_A + r_p)^3$

(c) $v_A r_A = v_p r_p$

(d) All of these

Ans. *(d) All of these*

Explanation: As it is given that the v_A is the velocity of the planet at Apogee and v_P is at Perigee. Now, by the law of conservation of angular momentum at both these points is:

$$\therefore \quad v_A r_A = v_P r_P$$

$$\therefore \quad r_A < v_P$$

This implies that time period is given by,

$$T^2 = \frac{4\pi^2}{GM} a^3$$

$$T^2 = \frac{4\pi^2}{GM}\left(\frac{r_p + r_A}{2}\right)^3 = \frac{\pi^2}{2GM}(r_p + r_A)^3$$

6. If a satellite is abruptly stopped in its orbit, which is a radius of the earth from the planet's surface, and allowed to fall freely into the earth. The speed at which it will strike the earth's surface will be:

(a) 7.919 m/s (b) 7.919 km/s

(c) 11.2 m/s (d) 11.2 km/s

Ans. *(b) 7.919 km/s*

Explanation: By conservation of mechanical energy.

$$-\frac{GMm}{2R} + 0 = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$v = \sqrt{\frac{GM}{R}} = 7.919 \text{ km/s}$$

Related Theory

➥ *Condition of weightlessness can be overcome by creating artificial gravity by rotating the satellite in addition to its revolution.*

7. If the earth stops moving around its polar axis, then what will be the effect on the weight of a body placed at the South Pole?

(a) Remains the same

(b) Increases

(c) Decreases but not zero

(d) Decreases to zero [Delhi Gov. QB 2022]

Ans. *(a) Remains the same*

Explanation: Variation in g due to the rotation of earth

$$g' = g - \omega^2 R\cos^2\lambda$$

At poles, $\lambda = 90°$ in the above expression, we get

$$g_{pole} = g - \omega^2 R\cos^2 90°$$

$$\therefore \quad g_{pole} = g$$

i.e., there is no effect of the rotational motion of the earth on the value of g at the poles.

8. A satellite's time span near the earth's surface is 1.4 hours. If it's at a distance of '4R' from the earth's center, calculate its time period.

(a) 32 hrs (b) $\left(\frac{1}{8\sqrt{2}}\right)$ hrs

(c) $8\sqrt{2}$ hrs (d) 16 hrs

Ans. *(c) $8\sqrt{2}$ hrs*

Explanation: As we know that

$$T^2 \propto r^3$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\frac{T_2}{1.4} = \left(\frac{4R}{R}\right)^{\frac{3}{2}} = 8$$

$$T_2 = 1.4 \times 8 = 8\sqrt{2} \text{ hrs}$$

9. Escape velocity for a projectile at the earth's surface is v_o. A body is projected from the earth's surface with velocity $2v_o$, the velocity of the body when it is at an infinite distance from the center of the earth is:

(a) v_o (b) $2v_o$

(c) $\sqrt{2}\, v_o$ (d) $\sqrt{3}\, v_o$

Ans. *(d)* $\sqrt{3}\, v_o$

Explanation: Apply energy conservation.

$$\Rightarrow \frac{-GM_e m}{R_e} + \frac{1}{2}m(2v_e)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{-GM_e m}{R_e} + \frac{1}{2}m\left(2\sqrt{\frac{2GM_e}{R_e}}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{3GM_e m}{R_e} = \frac{1}{2}mv^2$$

$$v'^2 = \frac{6GM_e}{R_e} = 3\times\frac{2GM_e}{R_e}$$

$$v' = \sqrt{3}\sqrt{\frac{2GM_e}{R_e}} = \sqrt{3}v_e$$

Related Theory

➥ *Escape velocity does not depend on the mass of the body being projected, the angle of projection or the direction of projection.*

10. A body attains a height equal to the radius of the earth when projected from the earth's surface. The velocity of the body with which it was projected is:

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{\frac{2GM}{R}}$

(c) $\sqrt{\frac{5}{2}\frac{GM}{R}}$ (d) $\sqrt{R}$

Ans. *(a)* $\sqrt{\frac{GM}{R}}$

Explanation: Apply energy conservation.

energy at surface = energy at the height

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{2R}$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{GM}{2R}$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}}$$

11. Two satellites of the same mass m orbiting around the earth (mass M) in the same orbit of radius r can collide since their rotational directions are opposing. The system's total mechanical energy (including satellites and earth) is (m << M):

(a) $-\frac{GMm}{r}$ (b) $-\frac{2GMm}{r}$

(c) $-\frac{GMm}{2r}$ (d) zero

Ans. *(a)* $-\frac{GMm}{r}$

Explanation: Total mechanical energy of satellite is.

$$E_A = E_p + E_K$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow v^2 = \frac{GM}{r}$$

Now
$$\text{T. E.} = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}\frac{mGM}{r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

So, the total energy of the system.

$$-\frac{GMm}{2r} - \frac{GMm}{2r} = -\frac{GMm}{r}$$

Related Theory

➥ *Gravitational potential energy or potential is a negative quantity whose maximum value is zero at infinite separation.*

12. A comet's maximum and minimum distances from the sun are 8×10^{12} m and 1.6×10^{12} m respectively. If its velocity is 60 m/s when it is closest to the sun, what is its velocity in m/s when it is farthest away?

(a) 12 (b) 60

(c) 112 (d) 6

Ans. *(a)* 12

Explanation: Apply angular momentum conservation law.

$$mv_1r_1 = mv_2r_2$$

$$\Rightarrow 60 \times 1.6 \times 10^{12} = v_2 \times 8 \times 10^{12}$$

$$v_2 = 12 \text{ m/sec}$$

13. Different points on earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the c.m. (centre of mass) causing translational and net torque at the c.m. causing rotation around an axis through the c.m. For the earth-sun system (approximating the earth as a uniform density sphere)

(a) the torque is zero

(b) the torque causes the earth to spin

(c) the rigid body result is not applicable since the earth is not even approximately a rigid body.

(d) the torque causes the earth to move around the sun.

[NCERT Exemplar]

Ans. *(a) the torque is zero*

Explanation: The torque on earth due to gravitational attractive force on earth is zero because the direction of force ($\vec{F}$). (gravitational) and the line joining ($\vec{r}$) the point of application of force (which also at c.m. of the earth) is along same line so the angle between $\vec{r}$, and $\vec{F}$ is zero so by.

$$\vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin 0° = 0$$

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

14. Assertion (A): A person feels weightlessness in an artificial satellite of the earth. However, a person on the moon (natural satellite) feels his weight.

Reason (R): Artificial satellite is a freely falling, body and on the moon's surface, the weight is mainly due to the moon's gravitational attraction.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: In artificial satellites, there is no gravitation but on the surface of the moon gravity is present with its effect very little in amount. That is why people feel weightlessness due to the absence of gravity on the artificial satellite and have some weight on the surface of the moon.

Weightlessness is a phenomenon when the gravitational force is completely used up for providing centripetal acceleration.

15. Assertion (A): An astronaut in an orbiting space station above the earth experiences weightlessness.

Reason (R): An object moving around the earth under the influence of the earth's gravitational force is in a state of free fall.

[Delhi Gov. QB 2022]

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The astronaut's typical force aboard the orbiting space station is zero.

As a result, the apparent weight of an astronaut aboard an orbiting space station is zero. An astronaut is said to be in a condition of weightlessness.

This is due to the fact that both the astronaut and the spacecraft are free-falling bodies.

So, both assertion and the reason are correct and reason is the correct explanation for assertion.

16. Assertion (A): Angular momentum of a satellite remains conserved.

Reason (R): Conservation of linear momentum leads to conservation of angular momentum.

Ans. *(c) A is true but R is false.*

Explanation: The only force acting on the satellite is the force of gravitational attraction which points directly toward the center of mass of the primary, the torque caused by this force equals zero, and therefore angular momentum of a satellite in an elliptical orbit is conserved about the center of mass of the primary. Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

17. Assertion (A): A body kept inside a spherical shell does not experience any gravitational force.

Reason (R): The body inside a spherical shell is protected from the gravitational attraction outside the shell.

Ans. *(d) A is false and R is also false*

Explanation: Inside the shell, the gravitational force is zero. This is because is no mass inside, the gravitational field is zero, thereby is zero. However, unlike a metallic shell which shield electrical forces, the shell does not shield other bodies outside it from exerting gravitational forces on a particle inside.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

18. *The satellite stays in orbit because it still has momentum energy it picked up from the rocket pulling it in one direction. Earth's gravity pulls it in another direction. This balance between gravity and momentum keeps the satellite orbiting around Earth. Satellites that orbit close to Earth feel a stronger tug of Earth's gravity. To stay in orbit, they must travel faster than a satellite orbiting farther away. The International Space Station orbits about 250 miles above the Earth and travels at a speed of about 17,150 miles per hour.*

(A) A rocket is launched into a circular orbit around the Earth. What more speed should be given to the spaceship now in order for it to overcome the earth's gravitational pull?

(B) An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. (i) Determine the height of the circular orbit around the earth's surface. (ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely on the earth, find the speed with which it hits the surface of the earth. Given M = mass of the earth and R = radius of earth.

(C) An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting the earth has a large size, can he hope to detect gravity?

Ans. (A) Let ΔK be the additional kinetic energy imparted to the spaceship to overcome the gravitational pull then,

$$\Delta K = -\text{(total energy of spaceship)} = \frac{GMm}{2R}$$

Total kinetic energy

$$= \frac{GMm}{2R} + \Delta K$$

$$= \frac{GMm}{2R} + \frac{GMm}{2R}$$

$$= \frac{GMm}{R}$$

then $\frac{1}{2}mv_2^2 = \frac{GMm}{R}$

$\Rightarrow$ $v_2 = \sqrt{\frac{2GM}{R}}$

But $v_1 = \sqrt{\frac{GM}{R}}$.

so additional velocity required

$$= v_2 - v_1$$

$$= \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$

$$= (\sqrt{2}-1)\sqrt{\frac{GM}{R}}$$

Alternate Solution:

Additional velocity

= escape velocity − orbital velocity

$$= v_{esc} - v_0$$

$$= \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$

$$= (\sqrt{2}-1)\sqrt{\frac{GM}{R}}$$

(B) (i) Let height above the earth's surface = h then

$$v_{orbital} = \sqrt{\frac{GM}{R+h}}$$

$$= \frac{1}{2}v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$\Rightarrow$ $R + h = 2R$

$\Rightarrow$ $h = R$

(ii) If the satellite is stopped suddenly, then its total energy

$$E_1 = -\frac{GMm}{2R}$$

Let its speed be v when it hits the earth's surface than its total energy on earth surface

$$E_2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

Conservation law for mechanical energy yield v,

$$E_1 = E_2$$

$$\Rightarrow \quad \frac{-GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

(C) An Astronaut inside a small spaceship experiences a very small negligible constant acceleration and hence the astronaut feels weightlessness. If the space station has too much mass and size, then it can experience acceleration due to gravity, *e.g.*, on the moon.

19. If we throw a ball vertically upwards from the surface of the earth; it rises to a certain height and falls back. If we throw it at a greater velocity, it rises to a greater height. If we throw it with suffiicient velocity, it may never come back. It will escape from the gravitational pull of the earth. This minimum velocity is called escape velocity. **[Delhi Gov. QB 2022]**

(A) Escape speed of a body of mass *m* depends upon its mass as:

(a) m^0 (b) m

(c) m^2 (d) m^3

(B) The escape velocity for an object projected vertically upward from the earth's surface is approx. 11 km/s. If the body is projected at an angle of 45° with the vertical, then the escape velocity will be:

(a) $\frac{11}{\sqrt{2}}$ km/s

(b) 11 km/s

(c) $11\sqrt{2}$ km/s

(d) 22 km/s

(C) The value of escape velocity on a certain planet is 2 km/s. Then the value of orbital speed of a satellite orbiting close to its surface is:

(a) 12 km/s (b) 1 km/s

(c) $\sqrt{2}$ km/s (d) $2\sqrt{2}$ km/s

(D) The escape speed of the planet is *v*. If the radius of the planet contracts to $\frac{1}{4}$ th of the present value, without any change in mass, the escape speed would become:

(a) halved

(b) doubled

(c) quadrupled

(d) one fourth

(E) The moon has no atmosphere as:

(a) The escape speed on the moon is very large as the thermal speed of the molecules of gases on the moon.

(b) The escape speed on the moon is equal to the thermal speed of the gaseous molecules on the moon.

(c) The escape speed on the moon is very small as compared to the thermal speed of the molecules of gases on moon.

(d) Size of the moon as compared to the earth is very less and hence escape speed of the moon is large.

Ans. (A) *(a)* m^0

Explanation: Escape speed of a body from Earth's surface is given by: $V_{mm} = \sqrt{2gR}$

As we can see from the above equation, there is no 'mass' term. Implies it can be written as m^0

So, the escape speed of a body is independent of its mass.

(B) *(b) 11 km/s*

Explanation: Escape speed of a body from Earth's surface is given by: $V_{mm} = \sqrt{2gR}$

This expression is obtained by conservation of energy and doesn't involve in which direction the body is thrown/projected.

So, irrespective of the angle of projection, escape speed of the body from Earth's surface remains constant *i.e.* ≈ 11 km/s

(C) *(c)* $\sqrt{2}$ km/s

Explanation: $v_e = 2\ kms^{-1}$

$v_0 = ?$

The orbital speed can be written in terms of escape velocity.

$$v_0 = \frac{v_e}{\sqrt{2}}$$

v_0 is the orbital speed for a satellite orbital close to its surface.

v_e is the escape velocity.

$$v_0 = \frac{v_e}{\sqrt{2}}$$

$$v_0 = \frac{2}{\sqrt{2}} = \sqrt{2}\ kms^{-1}$$

(D) (b) doubled

Explanation: Escape velocity on a planet is

$$v = \sqrt{\frac{2GM}{R}}$$

Now the radius of plant contracts to one fourth of present value without change in its mass.

So $\qquad R = \frac{R}{4}$

So new escaplee velocity

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{4 \times 2GM}{R}} = 2V$$

(E) (c) *The escape speed on the moon is very small as compared to the thermal speed of the molecules of gases on the moon*

Explanation: The escape velocity (the minimum velocity with which a body is to be projected so as to escape from the gravitational pulls on the surface of moon is very much less than the rms velocity of the molecules of gas at the surface temperature of moon. Therefore, the molecules will escape and therefore moon cannot hold an atmosphere.

VERY SHORT ANSWER Type Questions (VSA)

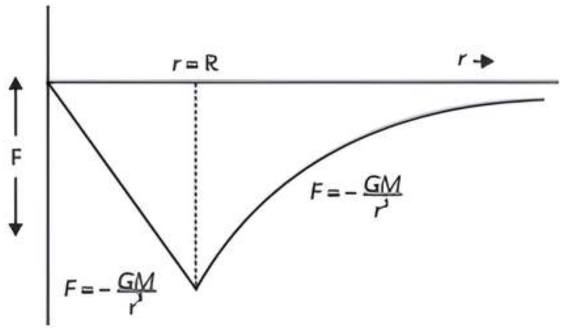
[1 mark]

20. **At a separation of the surface of Earth (r = 6400 km) gravity wants to pull the test mass closer and closer. Its gravitational potential energy wants to pull it closer and closer. To do the opposite (*i.e.*, move a test mass from a point to infinity) we need positive amounts of energy (like when we lift something up onto a shelf or shoot a rocket out into space really far). But gravitational potential energy wants to do the opposite. Why does gravitational potential energy possess this quality?**

Ans. Because a negative amount of work is done to bring an object closer to the Earth, gravitational potential energy is always a negative number when using this reference point. A satellite of mass *m* is launched from the surface of the Earth into an orbit of radius, where R_o is the radius of the Earth.

21. **The larger a planet or moon's gravity well is, the more energy it takes to achieve escape velocity and blast a ship off of it which means are we living at the bottom of a gravitational well. Give a reason.**

Ans. Yes, we are living at the bottom of a gravitational well. Figure shows the variation of gravitational force F with distance *r* from the centre of the earth. The graph has a force minimum at the surface of the earth (*r* = R).



22. **The radii of the two planets are R and 2R respectively and their densities ρ and ρ/2 respectively. What is the ratio of acceleration due to gravity at their surfaces?** **[Delhi Gov. QB 2022]**

Ans. Here,

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \cdot \frac{4}{3}\pi R^3 \rho$$

or $\qquad g \propto R\rho$

$$\frac{g_1}{g_2} = \frac{R\rho}{2R\frac{\rho}{2}}$$

$$= 1 : 1$$

23. **Why does an astronaut experience weightlessness in a spaceship?**

Ans. An astronaut orbiting the Earth does feel weightless because there is no ground or normal force to counteract the force of gravity. Thus, the astronaut is falling. However, since the astronaut is also moving forward superfast, he/she continually falls around the Earth rather than crashing into the Earth.

24. **How is the gravitational force between two point masses affected when they are dipped in the water keeping the separation between them the same?** **[NCERT Exemplar]**

Ans. By Newton's Universal law of gravitational force of attraction (F) between two bodies of masses m_1, m_2 separated by distance r is.

$$F = \frac{Gm_1m_2}{r_2}$$

G does not depend upon the medium. So, the force of attraction does not change if the masses are kept in water or any medium.

25. **The sun draws all living things on the planet. When the sun is directly below a body at midnight, it pulls in the same direction as the earth's draw on that body, when the sun is directly above a body at midday, it pulls in the opposite direction as the earth's pull. Then the body's weight will be higher at midnight than at noon.**

Ans. No, the earth is a satellite of the sun. Anybody placed on earth is also a satellite of the sun. The body and the earth both will have the same acceleration towards the sun. Hence, there will be no relative sun's gravitational acceleration between the body and the earth. That is, a body placed on earth will experience no gravitational effect due to the sun. It will experience a gravitational force only due to the earth. This will be the weight of the body measured on the earth and will remain the same for all twenty-four hours.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

26. **A projectile's route from the earth is generally parabolic, although it is elliptical for projectiles travelling to large heights. What is the reason for this?**

Ans. Under ordinary heights, the change in the distance of a projectile from the center of the earth is negligible compared to the radius of the earth. Hence, the projectile moves under a nearly uniform gravitational force and its path is parabolic. But for a projectile going to a very great height, the gravitational force decreases in inverse proportion to the square of the distance of the projectile from the center of the earth. Under such a variable force, the path of the projectile is elliptical.

27. **Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?**

Ans. Time taken by the Earth to complete one revolution around the Sun.

$$T_e = 1 \text{ year}$$

Orbital radius of the Earth in its orbit.

$$R_e = 1 \text{ AU}$$

Time taken by the planet to complete one revolution around the Sun.

$$T_P = \frac{1}{2}T_e = \frac{1}{2} \text{ year}$$

Orbital radius of the planet = R_p

From Kepler's third law of planetary motion, we can say that

$$\left(\frac{R_P}{R_E}\right)^3 = \left(\frac{T_P}{T_E}\right)^2$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{2/3}$$

$$\left(\frac{1/2}{1}\right)^{2/3} = (0.5)^{2/3} = 0.63$$

Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

28. **The radius of the circular orbits of two satellites, A and B of the earth, are 4R and R, respectively. If the speed of satellite A is 3v. What is the speed of satellite B?** **[NCERT Exemplar]**

Ans. Speed of satellite $v = \sqrt{\frac{GM}{r}}$

$$\Rightarrow \quad \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}} = 2$$

$$\Rightarrow \quad v_B = (3v)(2) = 6v$$

29. **A communication satellite from Earth that takes 24 hours to complete one circular orbit must eventually be replaced by a double-mass satellite. What is the ratio of the radius of the new orbit to the original orbit if the new satellite likewise has a 24-hour orbital time period?**

Ans. Time period of the revolution of satellite,

$$T = 2\pi\sqrt{\frac{R_e^3}{GM_e}}$$

where R_e and M_e are the radius of the orbit and mass of the earth. Thus, the time period doesn't depend on the mass of the satellite.

$$T^2 = \frac{4\pi^2}{GM}R_e^3$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{2}{3}} = \frac{r_1}{r_2} \qquad \left| \therefore T_1 = T_2 \right|$$

30. **An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting the earth has a large size, can he hope to detect gravity?** **[NCERT Exemplar]**

Ans. Astronaut inside a small spaceship experiences a very small negligible constant acceleration and hence astronaut feels weightlessness. If the space station has too much large mass and size then it can experience acceleration due to gravity *e.g.* on moon.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

31. **The earth launches a rocket towards the sun. The gravitational force on the rocket is zero at what distance from the earth's center? The sun's mass is 2×10^{30} kg, and the earth's mass is 6×10^{24} kg. Other planets and their effects are ignored.**

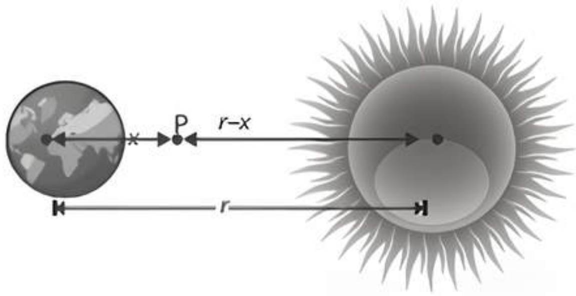
(Orbital radius is 1.5×10^{11} m).

Ans. Mass of the Sun, $M_s = 2 \times 10^{30}$ kg.

Mass of the Earth, $M_e = 6 \times 10^{24}$ kg.

Orbital radius, $r = 1.5 \times 10^{11}$ m,

Mass of the rocket = m



Let x be the distance from the center of the Earth where the gravitational force acting on Satellite P becomes zero.

From Newton's law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = \frac{GmM_e}{x^2}$$

$$\left[\frac{r-x}{x}\right]^2 = \frac{M_s}{M_e}$$

$$\frac{(r-x)}{x} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{1/2} = 577.35$$

$$1.5 \times 10^{11} - x = 577.35x$$

$$1.5 \times 10^{11} = 578.35\ x$$

$$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8}\text{m}$$

32. **A satellite orbits the earth at a height 'R' from the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence?**

[Delhi Gov. QB 2022]

Ans. The energy required to pull the satellite from the earth influence should be equal to the total energy with which it is revolving around the earth.

The K.E. of satellite = $\frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{R+h}$

$$\therefore \qquad v = \sqrt{\frac{GM}{R+h}}$$

The P.E. of satellite = $-\frac{GMm}{R+h}$

$$\text{T.E.} = \frac{1}{2}\frac{mGM}{(R+h)} - \frac{GMm}{(R+h)}$$

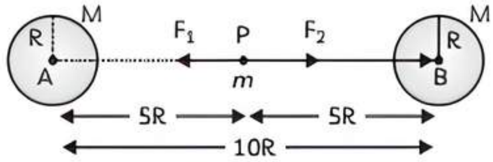
$$= -\frac{1}{2}\frac{GMm}{(R+h)}$$

$\therefore$ Energy required will be $\left(+\frac{1}{2}\frac{GMm}{(R+h)}\right)$

33. **Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid-point of the line joining their centres be in a stable equilibrium or unstable equilibrium? Give reason for your answer.**

[NCERT Exemplar]

Ans.



$m_1 = m_2 = M$ and $r = 10$ R

Let mass m is placed at the mid-point P of a line joining the centres of A and B sphere

$$|F_2| = |F_1| = \frac{GMm}{(5R)^2}$$

$$|F_2| = |F_1| = \frac{GMm}{25R^2}$$

As the direction of force, F_1 and F_2 are in opposite directions *i.e.*, equal and opposite forces are acting on m at P.

As net force $F_1 = -F_2$ or $F_1 + F_2 = 0$ is zero so the m is in equilibrium. If m is displaced x slightly from P to A

then PA = $(5R - x)$ and PB = $(5R + x)$

$$F_1 = \frac{GmM}{(5R-x)^2} \text{ and } F_2 = \frac{GmM}{(5R+x)^2}$$

$F_2 < F_1$ *i.e.*, resultant force acting on P is towards A.

Hence, equilibrium is an unstable equilibrium.

34. If different objects had different ratios of their inertial and gravitational masses, would they all accelerate at the same ratio in the gravitational field of the earth? Explain.

Ans. Suppose that at a point in the gravitational field of the earth, the value of acceleration due to gravity is g, if Mg is the gravitational mass of the body,

then gravitational force on the body = $M_G g$.

If M_1 is the inertial mass of the body, then the acceleration produced,

$$a = \frac{M_G g}{M_1}$$

It follows that if the ratio $\frac{M_a}{M_1}$ is different for different objects, then they would accelerate differently in the gravitational field of the earth. If it is so *i.e.*, the ratio $\frac{M_a}{M_1}$ is different for different objects. Then the time periods of the pendulum made of different materials (as aluminium, iron, lead etc.) would be different indicating that the pendulum made of different materials accelerate differently in the gravitational field of the earth.

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

35. A Star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity?

(Mass of the sun = 2×10^{30} kg).

Ans. Yes. A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $F_g = -\frac{GMm}{R^2}$.

Where,

M = Mass of the star = $2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$ Kg

m = Mass of the body

R = Radius of the star = 12 km = 1.2×10^4 m

$$F_g = \frac{6.67\times10^{-11}\times5\times10^{30}}{(1.2\times10^4)^2} = 2.31\times10^{11}\text{ mN}$$

Centrifugal force, $F_c = mr\omega^2$

Angular speed $\omega = 2\pi\nu$

ν = Angular frequency = 1.2 rev s^{-1}

$F_c = mr\,(2\pi\nu)^2$

$= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2$

$= 1.7 \times 10^5$ mN

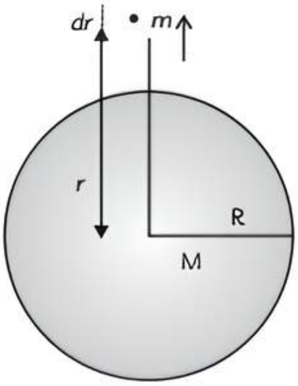
Since, $F_g > F_c$ the body will remain stuck to the surface of the star.

36. (A) Derive the expression for finding the escape velocity of the body.

(B) Does it depend on the location (e.g. different planets) from where it is projected?

[Delhi Gov. SQP 2022]

Ans. (A) Let a body of mass m be escape from the gravitational field of the earth. During the course of motion, let at any instant, the body be at a distance r from the center of the earth.



The gravitational force between the body and the earth is,

$$F = \frac{GMm}{r^2}dr$$

Work done to raise the body by distance dr is

$$dW = \frac{GMm}{r^2}dr$$

Total work done, W in raising the body from the surface of the earth to infinity is,

$$W = \int_R^\infty \frac{GMm}{r^2}dr = GMm\left[-\frac{1}{r}\right]_R^\infty$$

$$= GMm\left[-\frac{1}{\infty} + \frac{1}{R}\right]$$

$$= \frac{GMm}{R}$$

If we throw the body upward with a velocity v_o, then work done to raise the body from the surface of the earth to infinity is done by kinetic energy.

$$= \frac{GMm}{R} = \frac{1}{2}mv_e^2$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$v_e = \sqrt{2gR} \quad \text{—(i)}$$

Substituting the values in equation (i), we have $g = 9.81$ m/sec and $R = 6.4 \times 10^6$m

Escape velocity is given by,

$$v_e = \sqrt{9.81 \times 6.4 \times 10^6}$$
$$= 11200 \text{ m/s}$$
$$= 11.2 \text{ km/s}$$

(B) There is no relation of escape velocity with the location of projection. Hence, it does not depend on the location from where it is projected.

37. **A satellite is to be placed in equatorial geostationary orbit around earth for communication.**

(A) Calculate the height of such a satellite.

(B) Find out the minimum number of satellites that are needed to the cover the entire earth so that at least one satellite is visible from any point on the equator.

[$M = 6 \times 10^{24}$ kg, R = 6400 km, T = 24 *h*, $G = 6.67 \times 10^{-11}$ SI units]

[NCERT Exemplar]

Ans. (A) Mass of earth $M = 6 \times 10^{24}$ kg

Radius of earth $R = 6400$ km $= 6.4 \times 10^6$ m

Time period, $T = 24$ h $= 24 \times 3600$ s

$= 24 \times 36 \times 10^2$ s

$G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}

Orbital Radius = (R + h).

h is the height of the satellite from the earth surface.

$$v_0 = \sqrt{\frac{GM}{R+h}},$$

$$v_0^2 = \frac{GM}{(R+h)}$$

$$T = \frac{2\pi(R+h)}{v_0}$$

$$T^2 = \frac{4\pi^2(R+h)^2(R+h)}{GM}$$

$$T^2 = \frac{4\pi^2(R+h)^2}{GM}$$

$$\text{or} \quad (R+h) = \left[\frac{GT^2M}{4\pi^2}\right]^{1/3}$$

$$h = \left[\frac{GT^2M}{4\pi^2}\right]^{1/3} - R$$

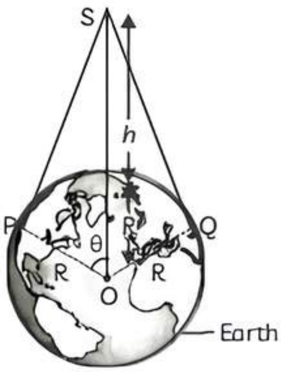
$$h = \left[\frac{6.67 \times 10^{11} \times (24 \times 36)^2 \times (10^2)^2 \times 6 \times 10^{24}}{4 \times 3.14 \times 3.14}\right]^{1/3}$$

$$h = x - 6.4 \times 106$$

$$\log x = \frac{1}{3}$$

$$\log N' = 27.8739$$
$$-\log D' = 4.9938$$
$$\log x = 7.6267$$
$$x = 4.2335 \times 10^7$$
$$h = 42.335 \times 10^6 - 6.4 \times 10^6$$
$$= 35{,}940 \text{ km}$$

(B) Let a satellite S is at the above of the earth's surface. Let angle subtended by satellite at centre of the earth O. Then in right angle ΔSPO



$$\cos\theta = \frac{R}{R+h} = \frac{R}{R\left[1+\frac{h}{R}\right]}$$

$$\cos\theta = \frac{1}{\left[1+\frac{h}{R}\right]}$$

(Height of geostationary satellite)
$R = 6.4 \times 10^6$ m

$$\cos\theta = \frac{1}{\left[1+\frac{h}{R}\right]}$$

$$\cos\theta = \frac{1}{\left[1+\frac{3.59\times10^7}{6.40\times10^6}\right]}$$

$$= \frac{1}{1+5.6} = \frac{1}{6.6}$$

$$\cos\theta = 0.1515$$

$$\theta = 81.28°$$

$$2\theta = [81.28 \times 2]$$

θ is covered by one satellite

Total angle to be covered = 360°

Number of satellites to cover

$$N = \frac{360°}{81.28\times2} = 2.21$$

So, the number of satellites to cover whole parts of the earth = 3.

38. Objects at rest on the earth's surface move in circular paths for a period of 24 hours. Are they in orbit in the same sense that an earth's satellite is in orbit? Why not? What would the length of the day have to be to put such objects in a true orbit?

Ans. The objects on the earth's surface are not in orbital motion w.r.t. the earth. So, that an object has an orbital motion close to the earth's surface, its orbital velocity, and period of motion must be,

$$v_0 = \sqrt{gR}$$

$$T = \frac{2\pi R}{v_0} = 2\pi\sqrt{\frac{R}{g}}$$

$$= 1.4 \text{ hrs}$$

Therefore, the length of the day has to be 1.4 hr in case the objects on the earth's surface are in true orbital motion like that of the earth's satellite.

NUMERICAL Type Questions

39. The gravitational potential difference between a point on the surface of a planet and a point 10 m above is 4 J/kg. Considering the gravitational field to be uniform, how much work is done in moving a mass of 2 kg from the surface to a point 5 m above the surface? (2m)

Ans. Gravitational field.

$$g = -\frac{\Delta V}{\Delta x} = -\left(\frac{-4}{10}\right) = \frac{4}{10}\ \frac{J}{kg-m}$$

Work done in moving a mass of 2 kg from the surface to a point 5 m above the surface.

$$W = mgh$$

$$= (2 \text{ kg})\left(\frac{4}{10}\frac{J}{kg-m}\right)(5m) = 4 \text{ J}$$

40. A body of mass m kg starts falling from a distance 2R above the earth's surface. What is its kinetic energy when it has fallen to a distance 'R' above the earth's surface? (Where R is the radius of the earth) (2m)

Ans. By conservation of mechanical energy,

$$-\frac{GMm}{3R} + 0 = -\frac{GMm}{2R} + K.E$$

$$\Rightarrow K.E = \frac{GMm}{R}\left[\frac{1}{2}-\frac{1}{3}\right] = \frac{1}{6}\frac{GMm}{R}$$

$$= \frac{1}{6}\frac{(gR^2)m}{R} = \frac{1}{6}mgR$$

41. One projectile after deviating from its path starts moving round the earth in a circular path radius equal to nine times the radius of earth R. What will be its time period? (3m)

Ans. Given:

$$r = 9R$$

As we know, time period is given by

$$T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad [\because GM = gR^2]$$

$$T = 2\pi\sqrt{\frac{r^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{(9R)^3}{gR^2}} \quad \text{(from question)}$$

$$T = 2\pi \times 9^{3/2}\sqrt{\frac{R}{g}}$$

$$T = 27 \times 2\pi\sqrt{\frac{R}{g}}.$$

42. **A black hole is a body from whose surface nothing may ever escape. What is the condition for a uniform spherical mass M to be a black hole? What should be the radius of such a black hole if its mass is the same as that of the Earth?**

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Ans. For a body to be a black hole, even light should not escape. So limiting escape velocity is 3×10^8 ms^{-1}

So, for the body of Mass M, the condition is,

$$\sqrt{\frac{2GM}{R}} \geq 3 \times 10^8 \text{ ms}^{-1}$$

For Earth, $M = 6 \times 10^{24}$ kg

$$\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{R} = (3 \times 10^8)^2 R$$

$$= 9 \times 10^{-2} \text{m}$$

$$R = 9 \text{ cm}$$